Mixed aleatoric-epistemic uncertainty for jet engine applications

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The way uncertainty is modeled affects the robustness evaluation

In the engineering uncertainty (aleatoric or epistemic) means

- Incomplete data
- Experience based information
- Model error and imprecision
- Random variability

Assumption

How would the robustness evaluation change if the lack of knowledge would be modeled?

Aleatoric versus epistemic uncertainty

Aleatoric Uncertainty

- **Randomness** related to the physical quantities or the system
- **Irreducible**
- **Data and probabilistic information** available

Epistemic Uncertainty

- **Incomplete knowledge** and/or lack of useful data
- **Reducible** (e.g. by gathering additional data, by refining models)
- **Interval information**, max-min, imprecise models

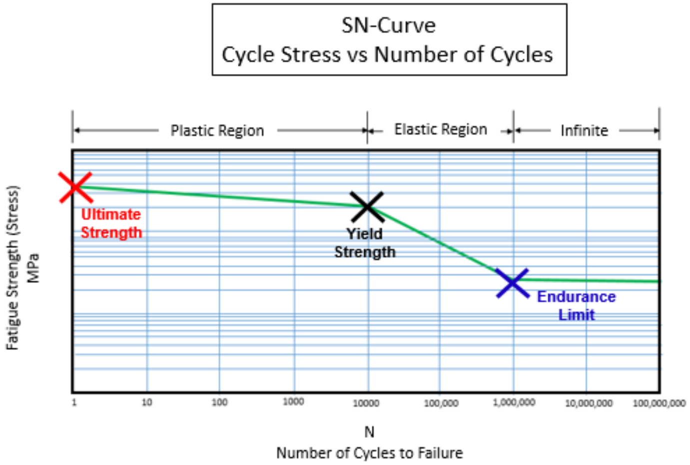
Different methods/approaches for different uncertainty types

Aleatoric	Epistemic	Mixed
Monte Carlo	Interval Analysis	Evidence Theory
Stochastic Collocation Methods	Bayesian Second Order Probability	Double Loop Approach

Example 1: from deterministic life models to possibilistic approach

Pure epistemic uncertainty quantification

- **Life tests** play a central role in the design of mechanical systems subjected to **cyclic loadings**
- **Experimental setup** is characterized by
  - ✓ Data becoming gradually available
  - ✓ **High variability** in the data



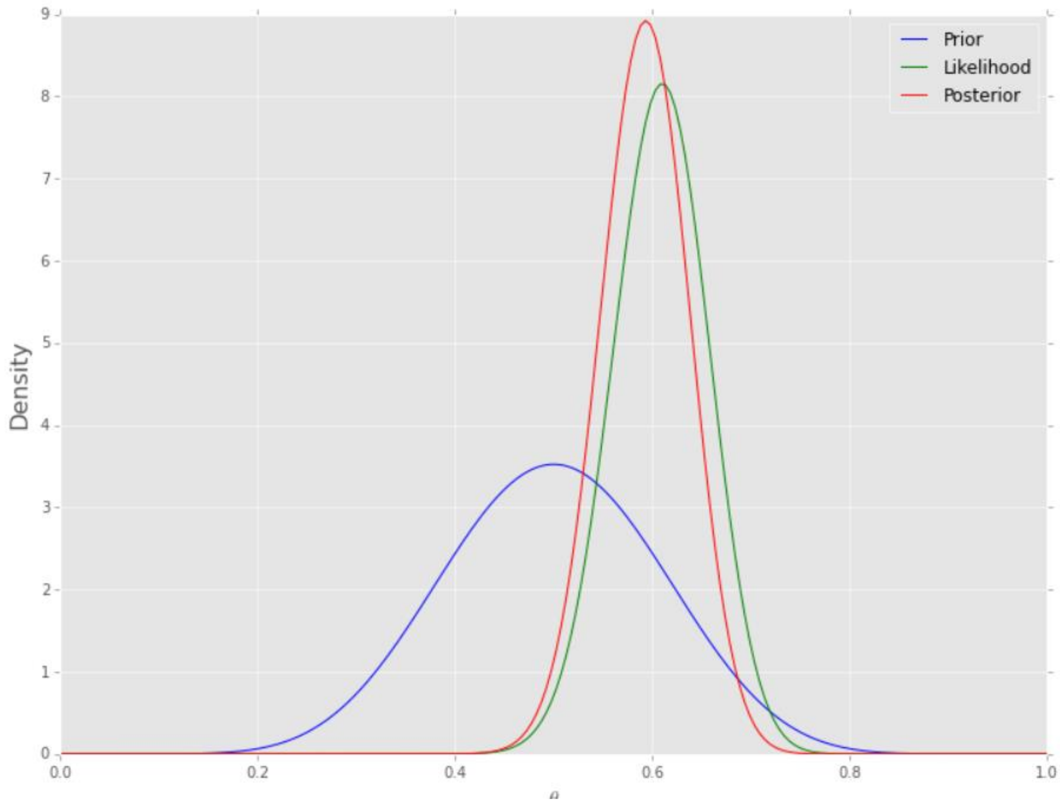
- The usual approach to estimate the life model parameters is **maximum likelihood estimation**
- No information on model parameter variability
- The **lack of knowledge** on model parameter can be addressed using **Bayesian methods**

Short overview on Bayesian probability theory

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- **Frequentist interpretation:** Bayes theorem is a **tool** to calculate the **probability of an event**, given that **another non-independent event has occured**
- **Bayesian interpretation:** probability is interpreted as a **measure of "degree of belief "**. Bayes rule links the degree of belief in an event A before and after accounting for evidence.
  - $P(A)$: the **prior**, the initial degree of belief in event A, before any data observation.
  - $P(A|B)$: the **posterior**, the degree of belief in A after accounting for the evidence B
  - $P(B|A)$: the **likelihood**, describes the compatibility of the evidence with the event A.

Graphical interpretation of Bayes theorem

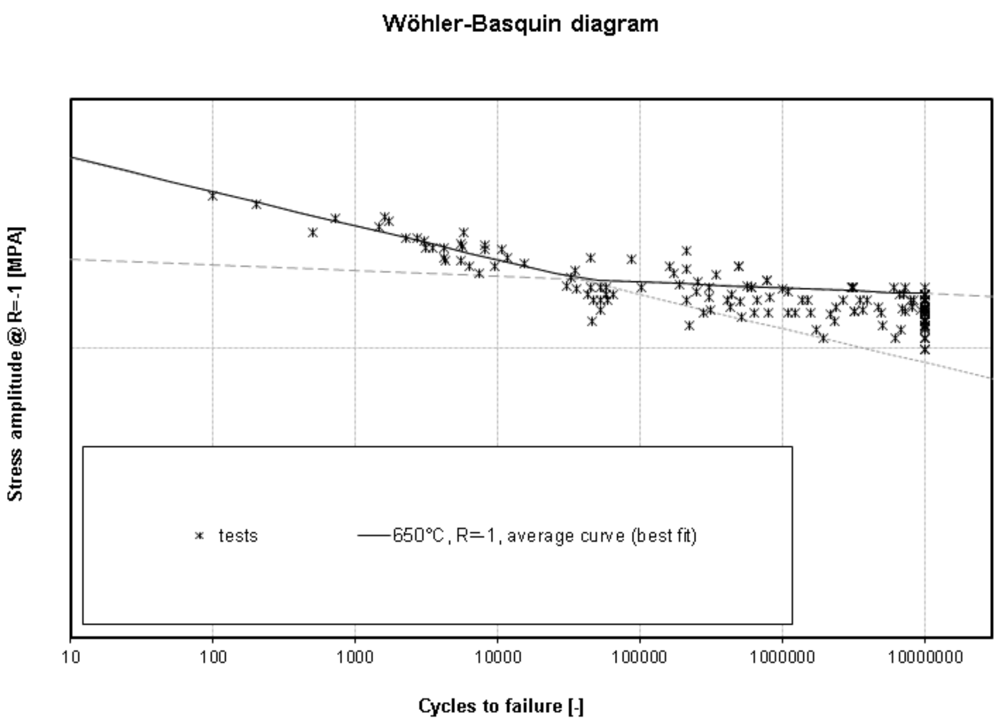


Computational Statistics with Python, Duke University lecture notes

Application: estimate parameter for fatigue life model

$$\underbrace{\sigma_{amp} \cdot \left(\frac{1-R}{2}\right)^{m-1}}_{\sigma_a{}^{walker}} = \left\{\left[A \cdot \left(2N_f\right)^b\right]^r + \left[C \cdot \left(2N_f\right)^d\right]^r\right\}^{1/r}$$

- Experimental data provided $\sigma_a{}^{walk,exp}$ and $N_f{}^{exp}$.
- Dataset including runouts.
- Parameters to be estimated: A, b, C, d, r



The uncertainty in the parameters can be modeled through priors

- **Uninformative** and **independent priors** for the parameters

$$A \sim U(0, 50000), C \sim U(0, 50000), d \sim U(-1, -0.01), r \sim U(0, 50)$$

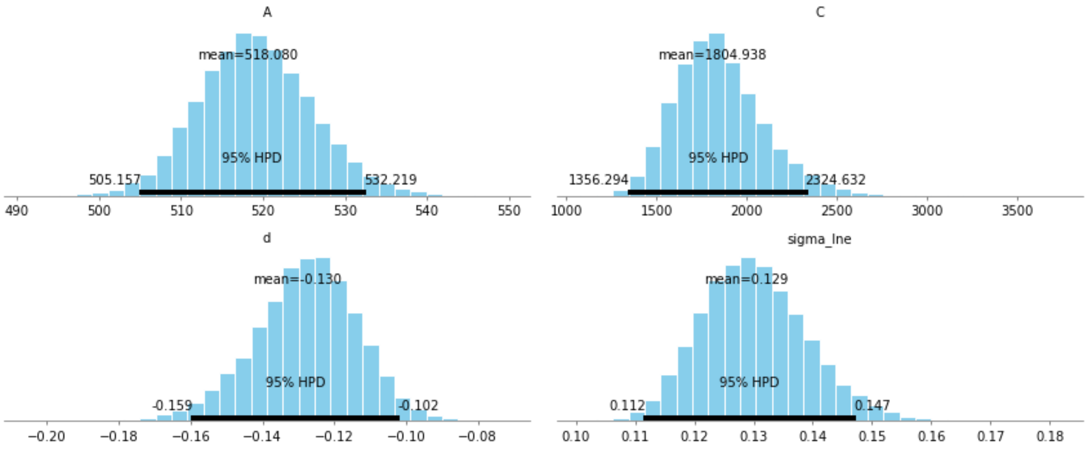
- **Uninformative prior** for the model error: $\sigma \sim U(0, \infty)$

- Available data: 105 Failures, 62 Runouts

- Bayesian calibration with Metropolis-Hastings algorithm

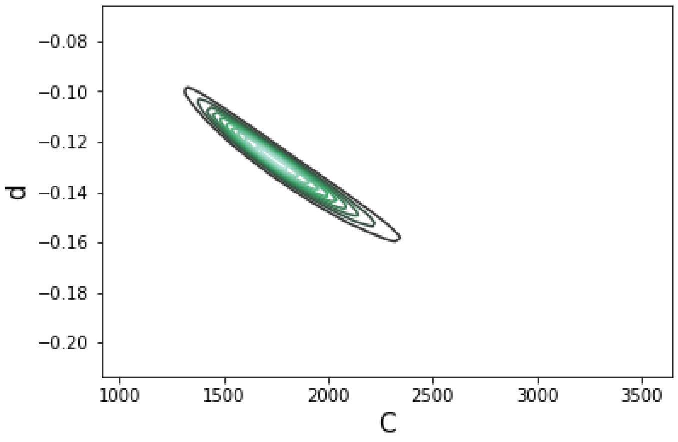
Bayesian calibration provides variability information on the parameters

Marginal Posterior Histograms



Correlations

	A	C	d	sigma_lne
A	1.000	-0.041	0.054	0.247
C	-0.041	1.000	-0.984	0.034
d	0.054	-0.984	1.000	-0.022
sigma_lne	0.247	0.034	-0.022	1.000



Add on in modeling lack of knowledge:

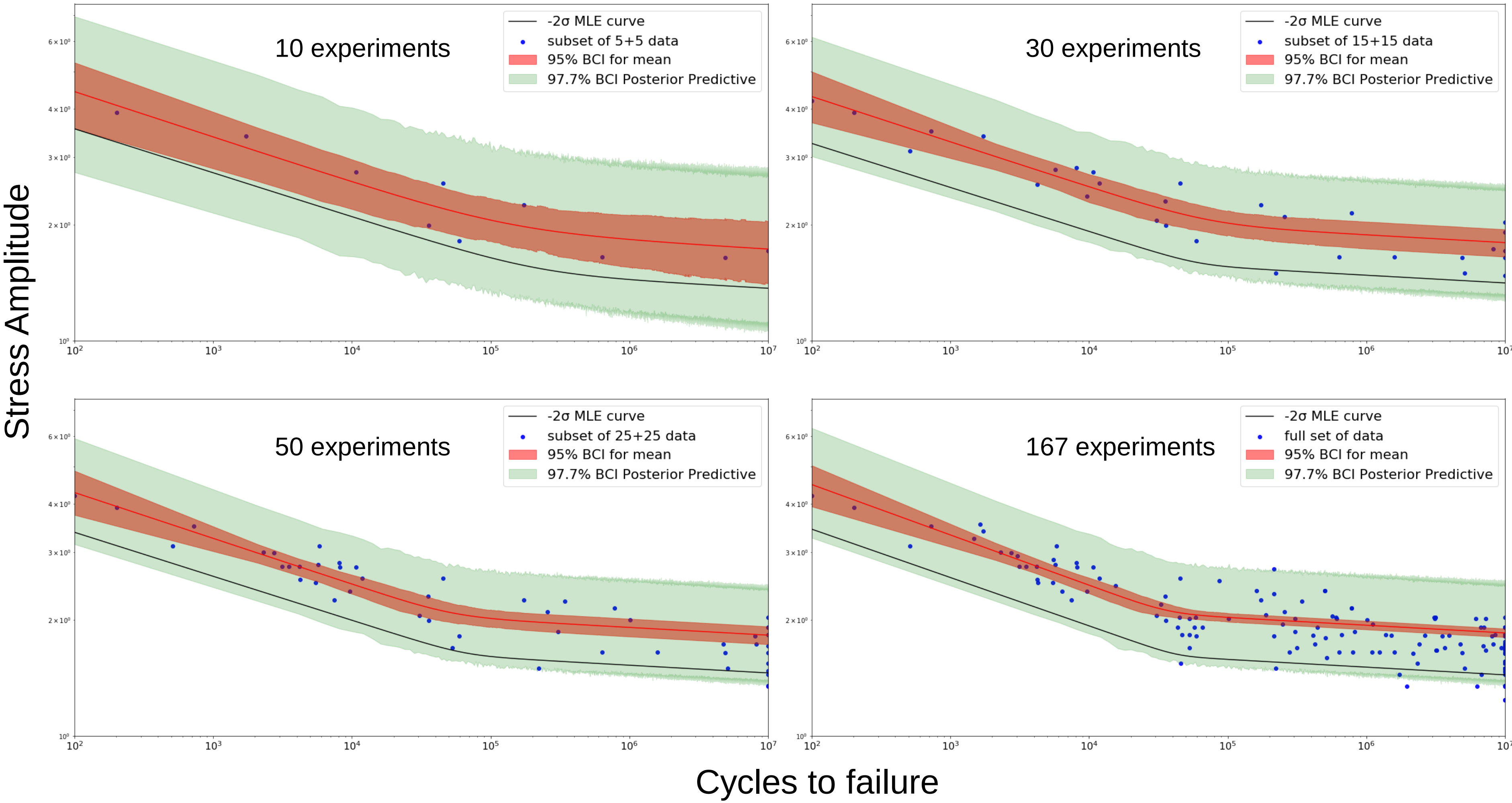
- More statistical information on parameter distribution
- Correlation information

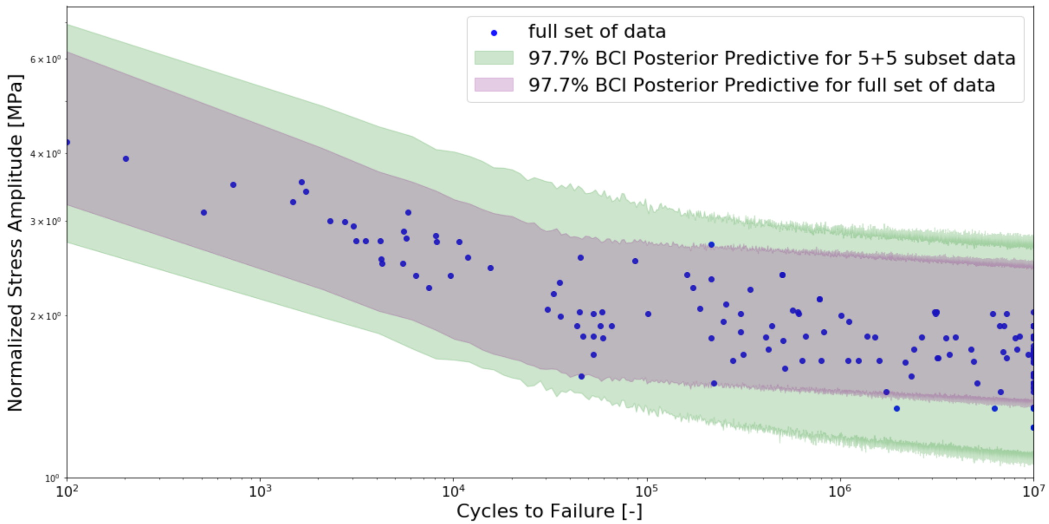
Does the number of data points affect the variance prediction?

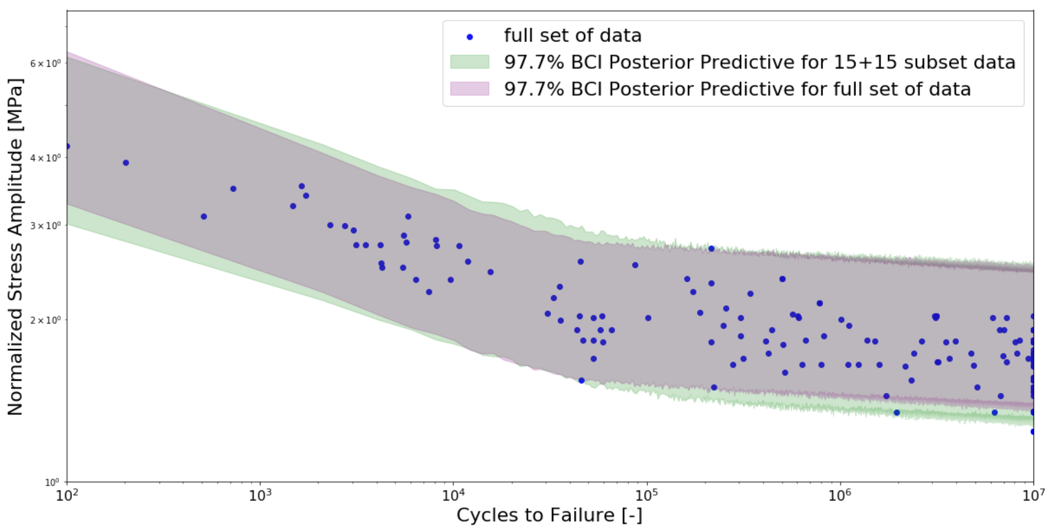
The original subset of data is subdivided in:

- ✓ 5 LCF and 5 HCF test data
- ✓ 15 LCF and 15 HCF test data
- ✓ 25 LCF and 25 HCF test data

Prediction of variance can be done by calculating the Bayesian Posterior Prediction Interval.


10 experiments
-2σ MLE curve
subset of 5+5 data
95% BCI for mean
97.7% BCI Posterior Predictive
30 experiments
-2σ MLE curve
subset of 15+15 data
95% BCI for mean
97.7% BCI Posterior Predictive
50 experiments
-2σ MLE curve
subset of 25+25 data
95% BCI for mean
97.7% BCI Posterior Predictive
167 experiments
-2σ MLE curve
full set of data
95% BCI for mean
97.7% BCI Posterior Predictive
Stress Amplitude
Cycles to failure





- 10 points are not sufficient to estimate the entire variability in the data set
- **At least 30 points should be provided.**

Conclusion

- Doing Bayesian analysis for fatigue data provides additional information on the probability distribution of model coefficients

- Using flat uniform prior distributions should be avoided and weakly informative priors should be used instead.

- The more runouts included in the dataset, the less informative the data is.

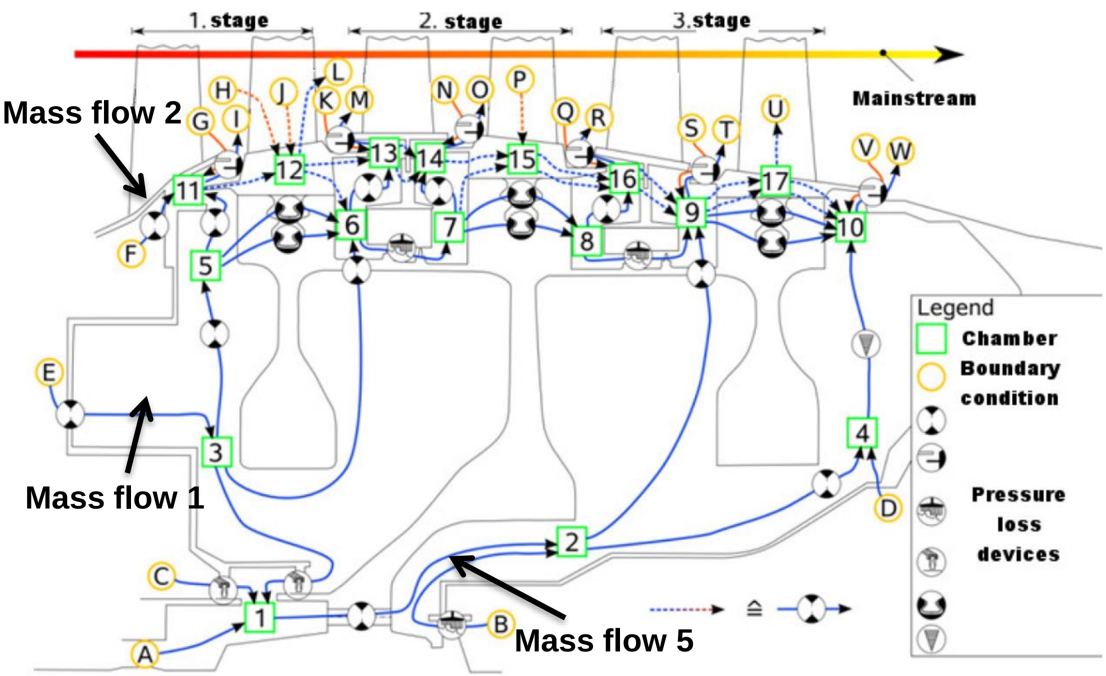
- Bayesian posterior predictive interval more conservative than -2σ MLE curve (without confidence interval)

Example 2: Manufacturing tolerance modeling in the Secondary Air System (SAS)

Mixed aleatoric - epistemic uncertainty quantification

- Model **input parameter uncertainty** the main focus.
- Only intervals available within which the true parameter value lies
- Late design phase → performing measurements is practically possible
- Focus on robustness

Jet engine secondary air system (SAS) is responsible for the cooling of the internal parts of the engine.



Input:

- Boundary conditions dependent on **performance parameters (pressure, temperature).**
- **Geometries** of the different flow elements.

Output:

- Pressure and temperature at each chamber (in green).
- Mass flow rate in each flow line (in blue).

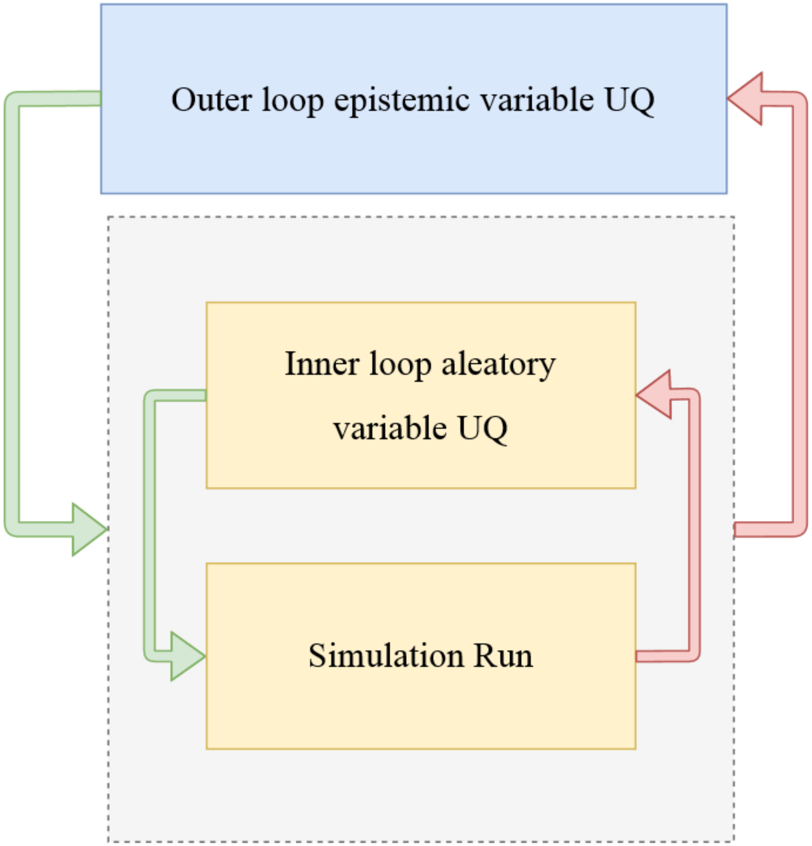
SAS uncertainties are mixed aleatoric and epistemic

Input Parameters	Uncertainty Source	Available Information	Categorization
Performance parameters	Ambient conditions Engine-to-engine variations Deterioration	Expert Information Legacy engines	Aleatory
Geometric variables (except rotor-stator gaps)	Manufacturing tolerances	Technical Drawings	Epistemic
Rotor-stator gaps	Manufacturing tolerances Engine operation	Technical Drawings -	Aleatory

12 aleatoric uncertain variables + 51 epistemic uncertain variables

The double loop approach conceptually separate epistemic and aleatoric uncertainty

$$Y = g(X), \textbf{with } \boldsymbol{X} = [\boldsymbol{X_a}, \boldsymbol{X_e}]$$



Dakota user's manual, Sandia National Laboratories.

$\boldsymbol{X_e}$ → interval information

Method:

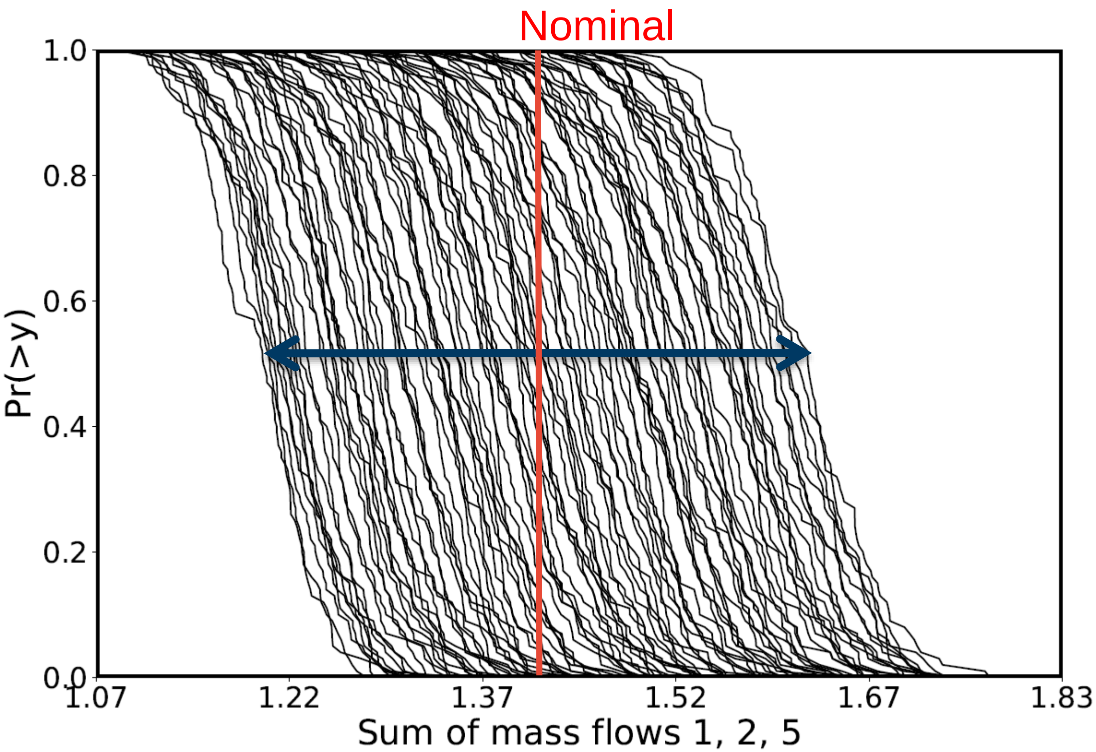
- IVP (interval value probability)

$\boldsymbol{X_a}$ → probability distribution

Method:

- Monte Carlo
- Stochastic collocation method

Aleatoric uncertainty = Shape of CCDF
Epistemic uncertainty = Interval width



- Narrowing tolerance on the geometric features of SAS means a reduction of interval width
- The **total amount of cooling air** has **less variability**
- The **tolerance** can be changed to produce an **improvement** with respect to **nominal**
- There is a **remaining variability** due to boundary conditions variation and labyrinth gap variation **which is irreducible**

Conclusion

- Parameter and model uncertainty should be modeled according to the available information
- Lack of knowledge can also be modeled through epistemic uncertainty
- **Bayesian methods for calibration** provides **additional information** useful for quantifying the **variability, e.g. in life models**
- Mixed aleatoric and epistemic uncertainty methods **separate the uncertainties both conceptually and computationally** and help obtain separately the influence of each uncertainty type

Thank you for your attention

Software

- **PyMC3**: Python library for Bayesian Statistical Modeling focusing on advanced Markov Chain Monte Carlo (MCMC) algorithms

  - Metropolis Hastings Algorithm (MH):
    - main oldest MCMC algorithm

  - No-U-Turn Sampler (NUTS):
    - State of the art MCMC algorithm which uses first order gradient information of the log-posterior density.
    - Handles only continuous parameters. Suitable in our case where we have an analytical likelihood function.