TECHNISCHE UNIVERSITÄT DRESDEN

UNCERTAINTY MANAGEMENT FOR ROBUST INDUSTRIAL DESIGN IN AERONAUTICS
UMRIDA

Faculty of Mechanical Science and Engineering, Institute of Fluid Mechanics, Chair of Turbomachinery and Jet-Propulsion

Sensitivity analysis of a two-stage high pressure compressor using an extended Latin hypercube sampling

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Probabilistic Analysis

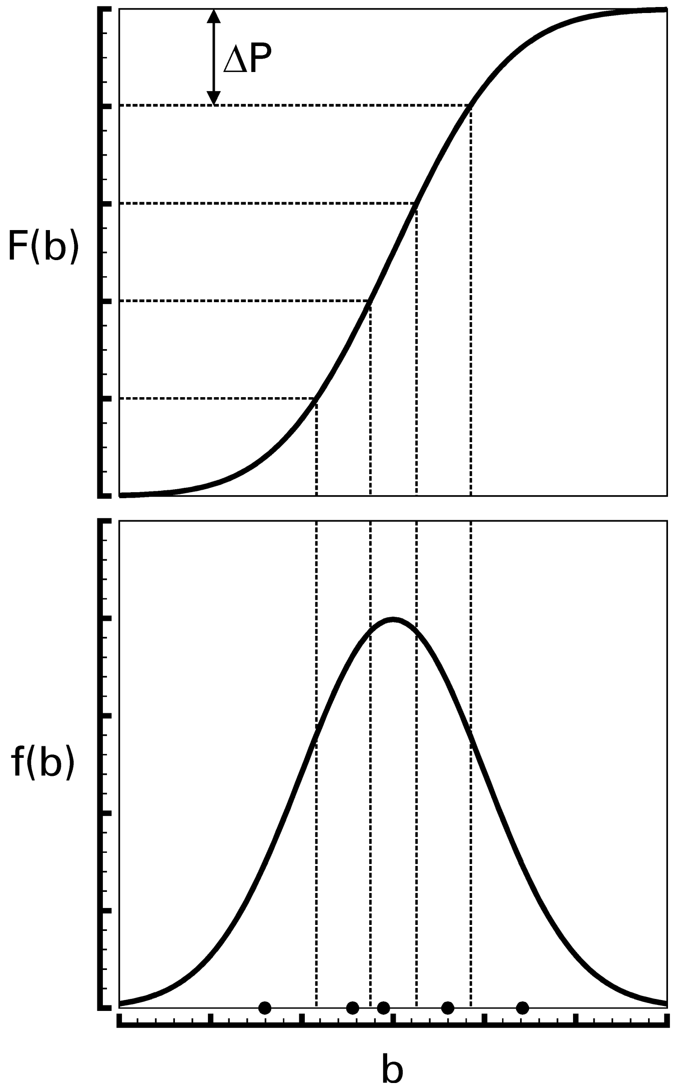
Inputs, **X** → **Model** $\mathcal{M}$ → Outputs, **Y**

time consuming deterministic Model, non-linear behavior in the outputs, 10+ input variables

IC09

→ sensitivity analysis with Monte Carlo methods using Latin Hypercube sampling → $\mathbf{n_{sim}}$**?**

Latin Hypercube Sampling



CHARACTERISTIC:

- each realization represents equal probability ΔP

APPROACH:

- define number of realizations n_{sim}
- determine $\Delta P=1/n_{sim}$ wide intervals on F(b)
- select one value at random from each interval

PROPERTIES:

- good representation of cdf with “few” realizations – variance reduction
- more stable analysis outcomes than random sampling
- easier implementation than stratified sampling methods
- mean value and distribution function can be estimated unbiased

extended Latin Hypercube Sampling

INITITAL POSITION

- define *group size* n_g and *level* l
- use “classic” LHS with $n_{sim,0} = n_g$ realizations

APPROACH

- Use “small” *group size* and reach the desired $n_{sim,N}$ by extension *level* times

IMPLEMENTATION

- reduplicate the intervals on F(b) if necessary
- per extension step only n_g values are added
- select one value at random from each free interval
- selection of the interval is based on D* as the largest negative distance between continuous and discrete cdf for each original interval

$$D^* = \min_{1 \le i \le n_{sim}} \left(\frac{i}{n_{sim}} - F(y_i) \right)$$

Correlation control

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Iterative Restricted Pairing[1]

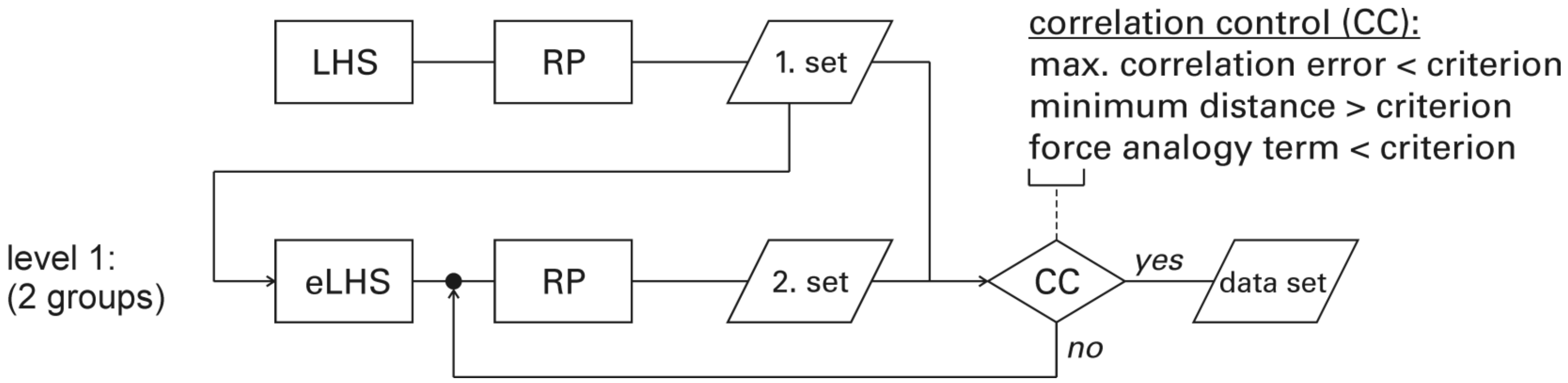
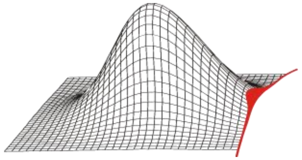


Figure 2: CC-Algorithm

[1] on the basis of: *Ramesh A. Dandekar, Michael Cohen, and Nancy Kirkendall. Sensitive micro data protection using latin hypercube sampling technique. In Inference Control in Statistical Databases, page 117–125. Springer, 2002.*

Evaluation of eLHS



generation of standard normal distributions

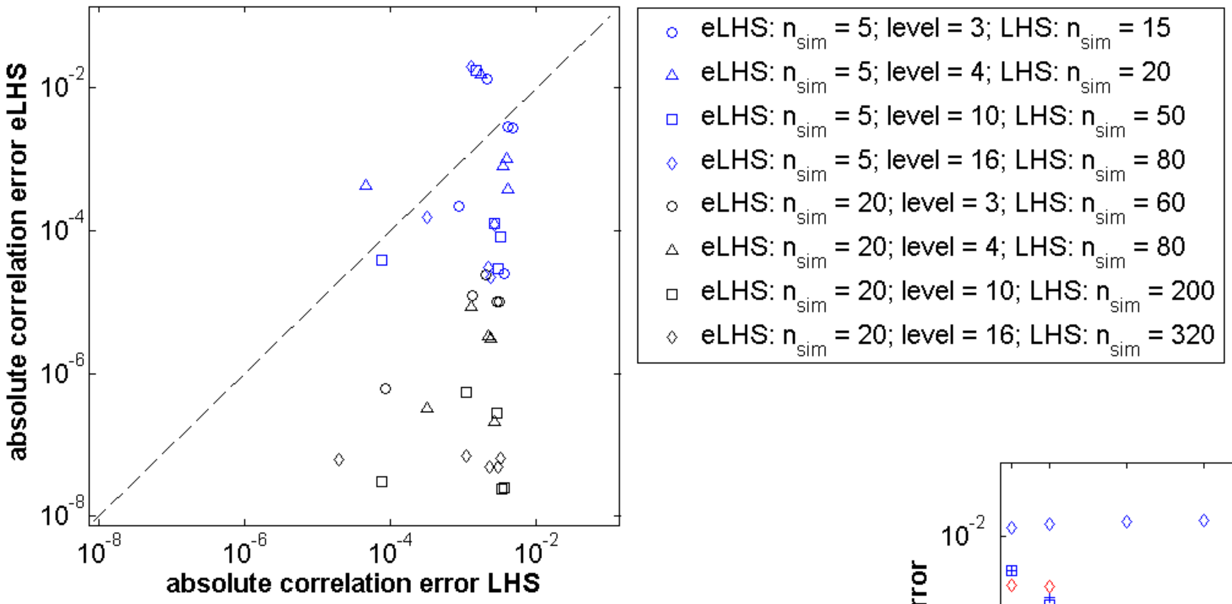
level	group size		
	5	10	20
3	15	30	60
4	20	40	80
6	30	60	120
8	40	80	160
10	50	100	200
12	60	120	240
13	65	130	260
16	80	160	320

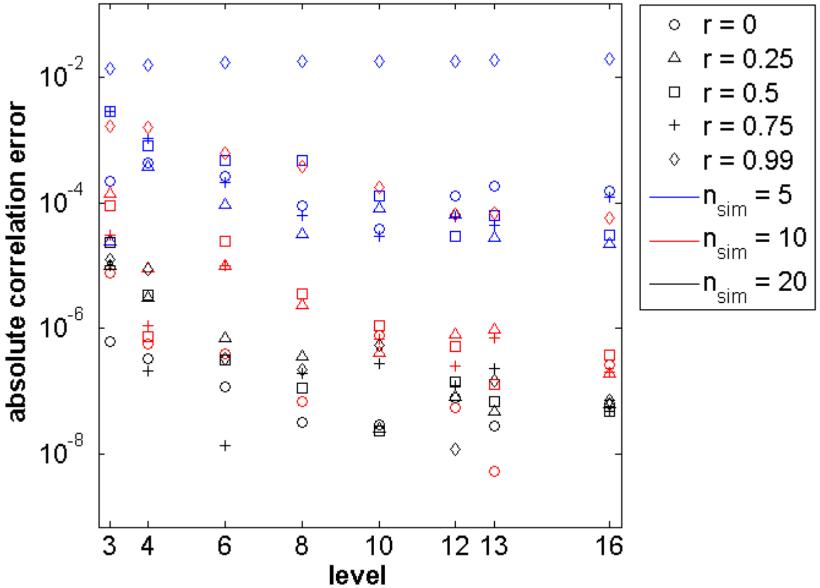
Table 1: Experimental matrix of eLHS

$$\tilde{r} = [0,0.25,0.5,0.75,0.99]^T$$

with 1000 repetitions and comparison with LHS of same size

Correlation of input variables



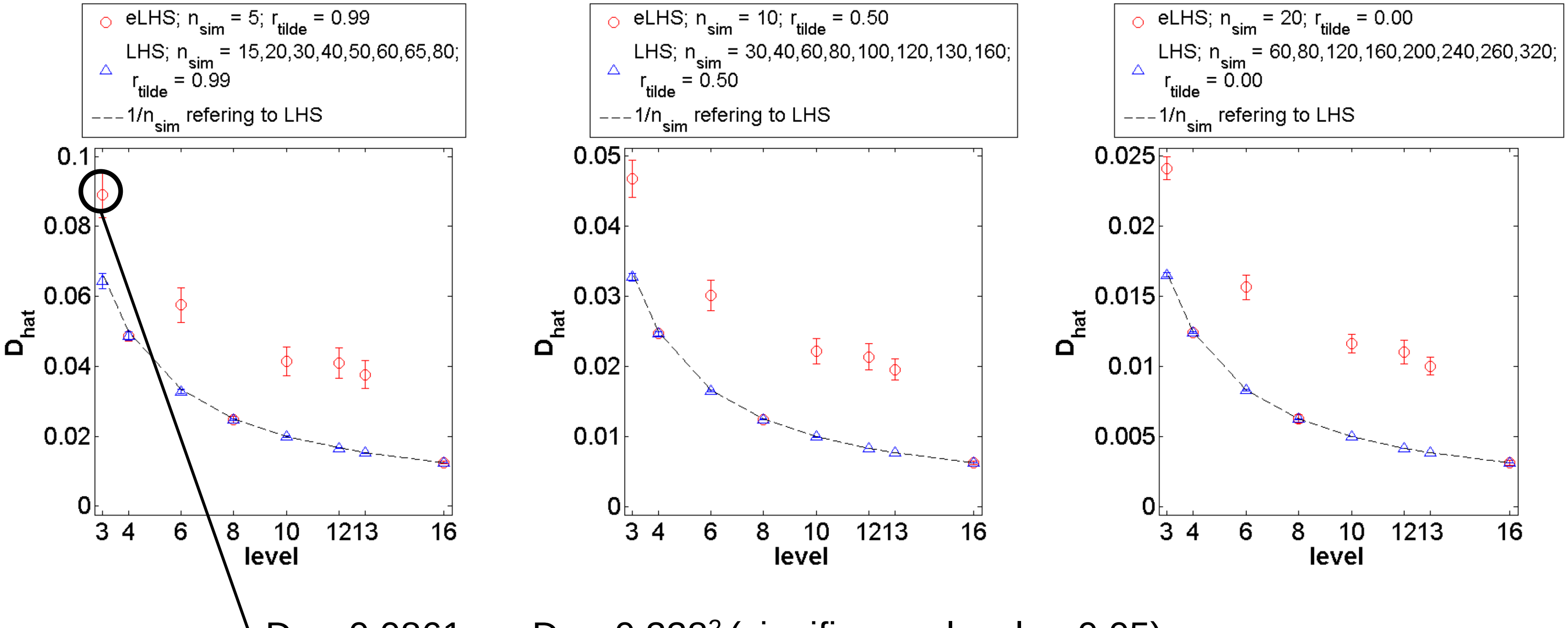


- iterative Restricted Pairing leads to very low mean absolute correlation errors of maximal 10^{-2} for LHS and eLHS
- correlation error of eLHS is in the majority of cases below that one of the LHS
- for high correlation values at low group size correlation control algorithm is not able to deliver the same performance for the eLHS as for the LHS with correspondingly l times higher number of realizations.

Reproduction of the distribution function

K-S-test of goodness of fit

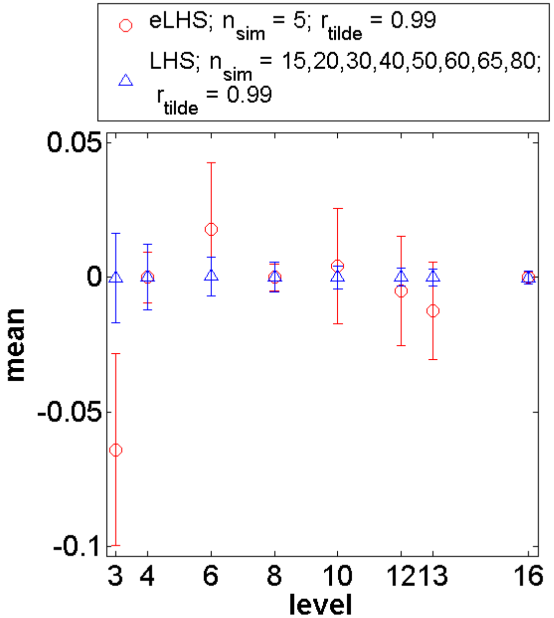
$$\hat{D} = \max_{1 \le i \le n_{sim}} \left| F(y_i) - \frac{i}{n_{sim}} \right|$$

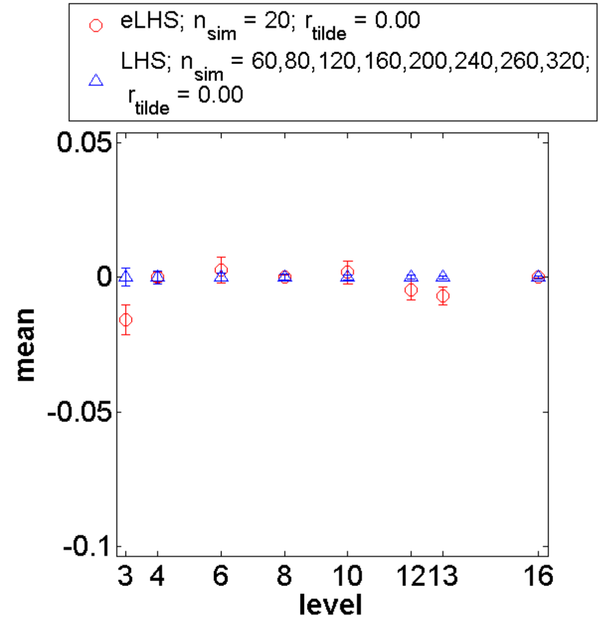


D_{hat}=0.0861 vs. D_{crit}=0.338[2] (significance level α=0.05)

[2] *L. Sachs and J. Hedderich. Angewandte Statistik. Springer, 13. edition, 2009*

Reproduction of mean value





$$D^* = \min_{1 \le i \le n_{sim}} \left(\frac{i}{n_{sim}} - F(y_i) \right)$$

$$|D^*| = |F(y_i) - 1| \quad with \;\; i = n_{sim}$$

- level 4, 8 and 16 lie closely one upon the other for eLHS and LHS
- characteristic shape of the deviations due to the allocation of intervals

Application of eLHS in Sensitivity analysis

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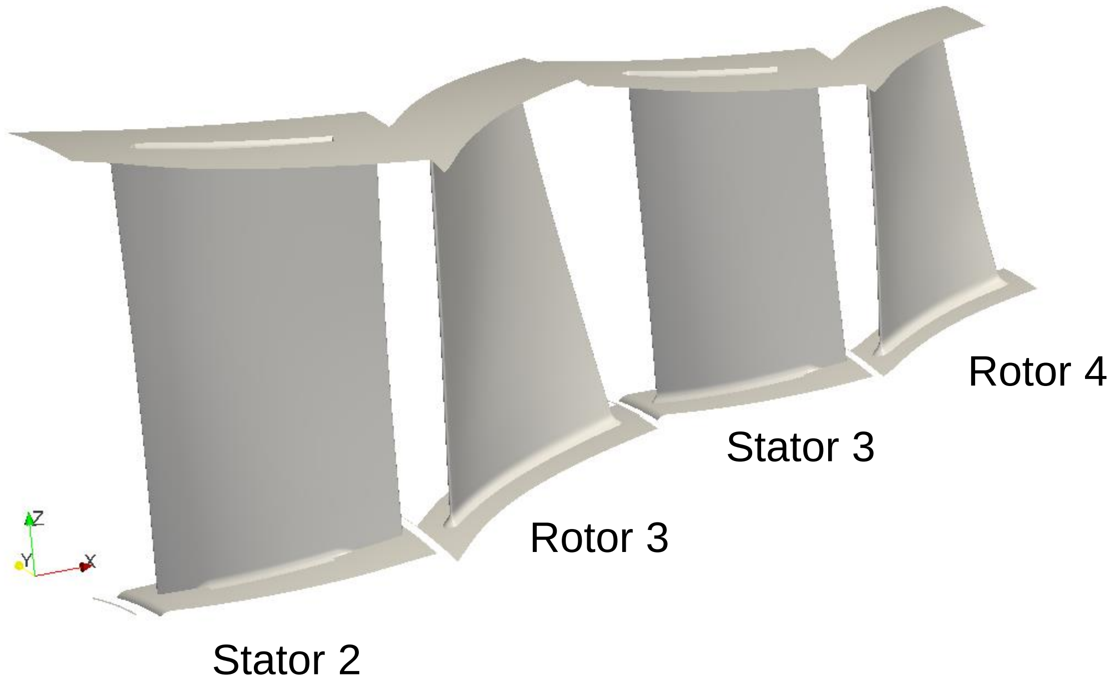
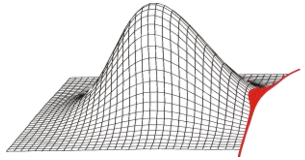
- test case IC09 delivered by Rolls-Royce Germany (RRD)
- resembles 2 stages of a typical high pressure compressor (hpc)
- boundary and initial conditions are given by radial profiles at inlet, fixed mass flow at the outlet
- data transfer between the blocks is done by mixing planes



Test case IC09



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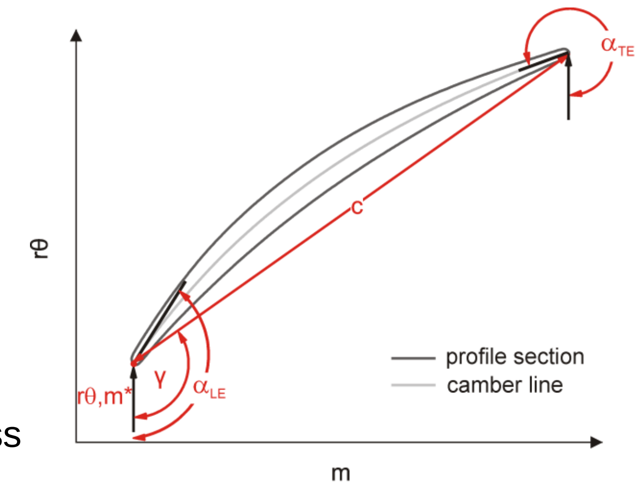
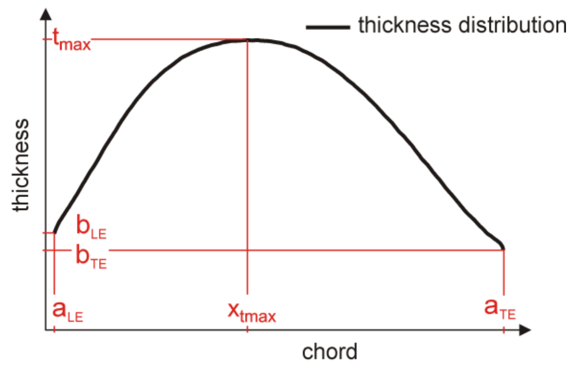
- geometric parameterization was done with the parameter model of Heinze et al.[3]; geometry variations with delta-parameter model of Lange et al.[4]
- one averaging section in spanwise direction is sufficiently accurate, see Lange et al.[5]

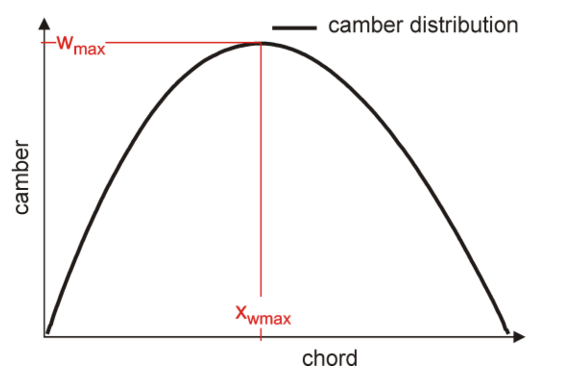
ax_{pos} – axial position leading edge point
tan_{pos} – tangential position leading edge point
c – chord length
γ – stagger angle
α_{LE} – angle at leading edge
α_{TE} – angle at trailing edge
t_{max} – maximum thickness
x_{tmax} – position of maximum thickness
w_{max} – maximum camber
x_{wmax} – position of maximum camber
a_{TE} – large semi axis trailing edge
b_{TE} – small semi axis trailing edge
a_{LE} – large semi axis leading edge
b_{LE} – small semi axis leading edge
fillet – fillet radius



*ax_{pos} and tan_{pos} in the cartesian coordinate system

further parameter:
fillet radius





Sensitivity analysis

- variation of Rotor 3 only
- always the same grid setup was used
- main characteristics of the MCS:

sampling method:	extended Latin Hypercube (eLHS)
correlation control:	iterative Restricted Pairing
shots:	n_g=30, level l=4

- setup, control and evaluation of the MCS with ProSi

Spearman rank correlation

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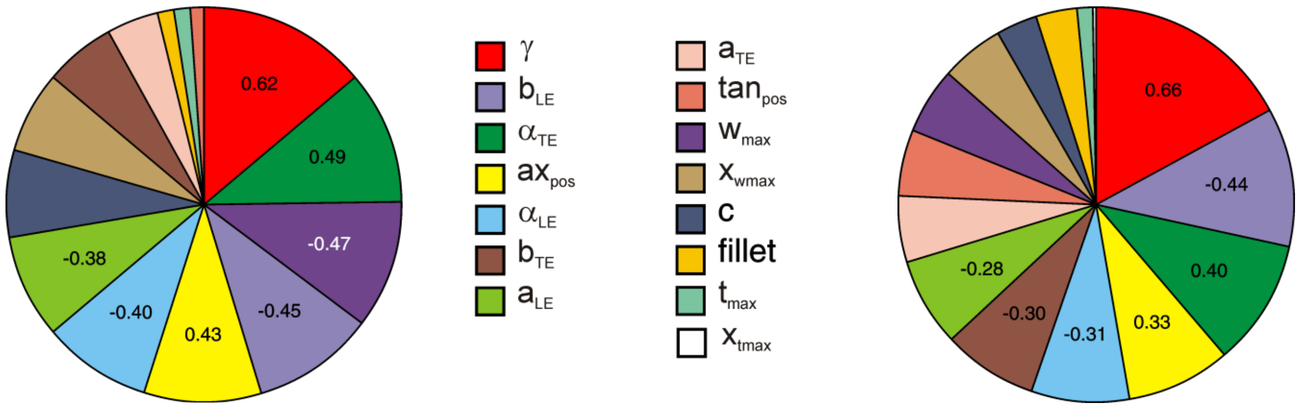
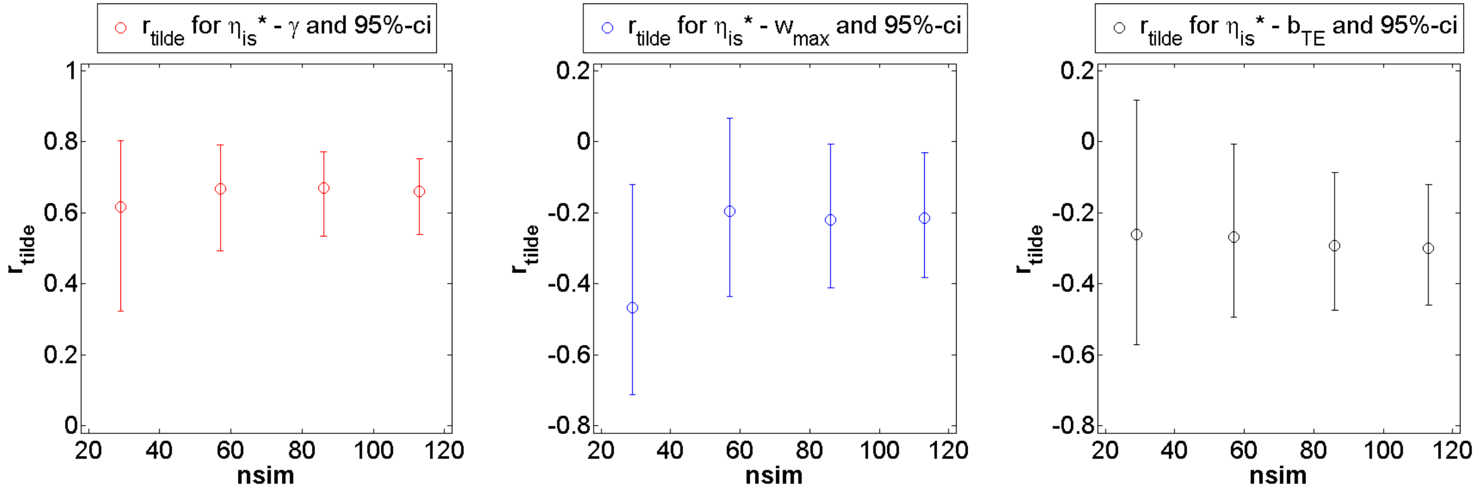
Isentropic compressor efficiency η_{is}* of the two-stage compressor





Coefficient of Importance (CoI)

- CoI is based on meta models and calculated with the Coefficient of Determination R^2

$$CoI_{ij} = R_j^2 - R_{ij}^2$$

- assessment of the quality of the response surface with cross-validation: Monte Carlo cross-validation (MCCV) by Beschorner[6] with splitting ratio of 0:85 and number of runs of 1000
- result quantity total pressure ratio π of the two-stage compressor
- approximation with a first order polynomial without mixed terms in each level

level	1	2	3	4
SCR	1:813	3:563	5:375	7:063
R^2	0:958	0:913	0:915	0:899
CoD_{MCCV}	0:748	0:834	0:876	0:865
average R^2 - CoD_{MCCV}	0:853	0:874	0:895	0:882

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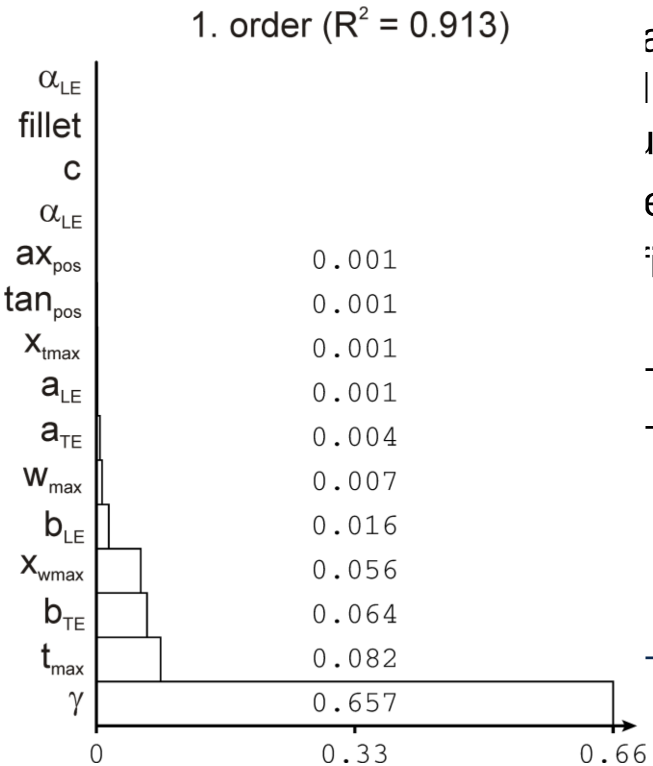

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Summary

- biggest advantages over LHS if extension is considered before the start of a probabilistic simulation
- method does not maintain the LHS design in each level
- a more variable extension is achieved compared to duplication of the realizations
- If at a certain level all intervals are occupied, the extended sample corresponds to a LHS
- Each extension represents an LHS design by itself
- application of iterative RP leads to low deviations from the target correlation for LHS and eLHS despite small number of realizations and high correlations
- with the sample extension method it is possible to use the statistical quality, e.g. confidence intervals, of certain statistical measures as a termination criterion
- extension results in an increased gain of information from a probabilistic analysis

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References

(1) A. Dandekar, Michael Cohen, and Nancy Kirkendall. Sensitive micro data protection using latin hypercube sampling technique. In Inference Control in Statistical Databases, page 117–125. Springer, 2002.

(2) L. Sachs and J. Hedderich. Angewandte Statistik. Springer, 13. edition, 2009

(3) Kay Heinze, Marcus Meyer, Jens Scharfenstein, Matthias Voigt, and Konrad Vogeler. A parametric model for probabilistic analysis of turbine blades considering real geometric effects. CEAS Aeronautical Journal, 5(1):41–51, 2014.

(4) Alexander Lange, Konrad Vogeler, Volker Gümmer, Henner Schrapp, and Carsten Clemen. Introduction of a parameter based compressor blade model for considering measured geometry uncertainties in numerical simulation. In ASME Turbo Expo 2009: Power for Land, Sea, and Air, page 1113–1123. American Society of Mechanical Engineers, 2009.

(5) Alexander Lange, Matthias Voigt, Konrad Vogeler, Henner Schrapp, Erik Johann, and Volker Gümmer. Probabilistic CFD simulation of a high-pressure compressor stage taking manufacturing variability into account. In ASME Turbo Expo 2010: Power for Land, Sea, and Air, page 617–628. American Society of Mechanical Engineers, 2010.

(6) André Beschorner, Matthias Voigt, and Konrad Vogeler. Monte carlo cross-validation for response surface benchmark (to be published). In Proceedings of the 12th International Probabilistic Workshop, Weimar, 2014. International Probabilistic Workshop.