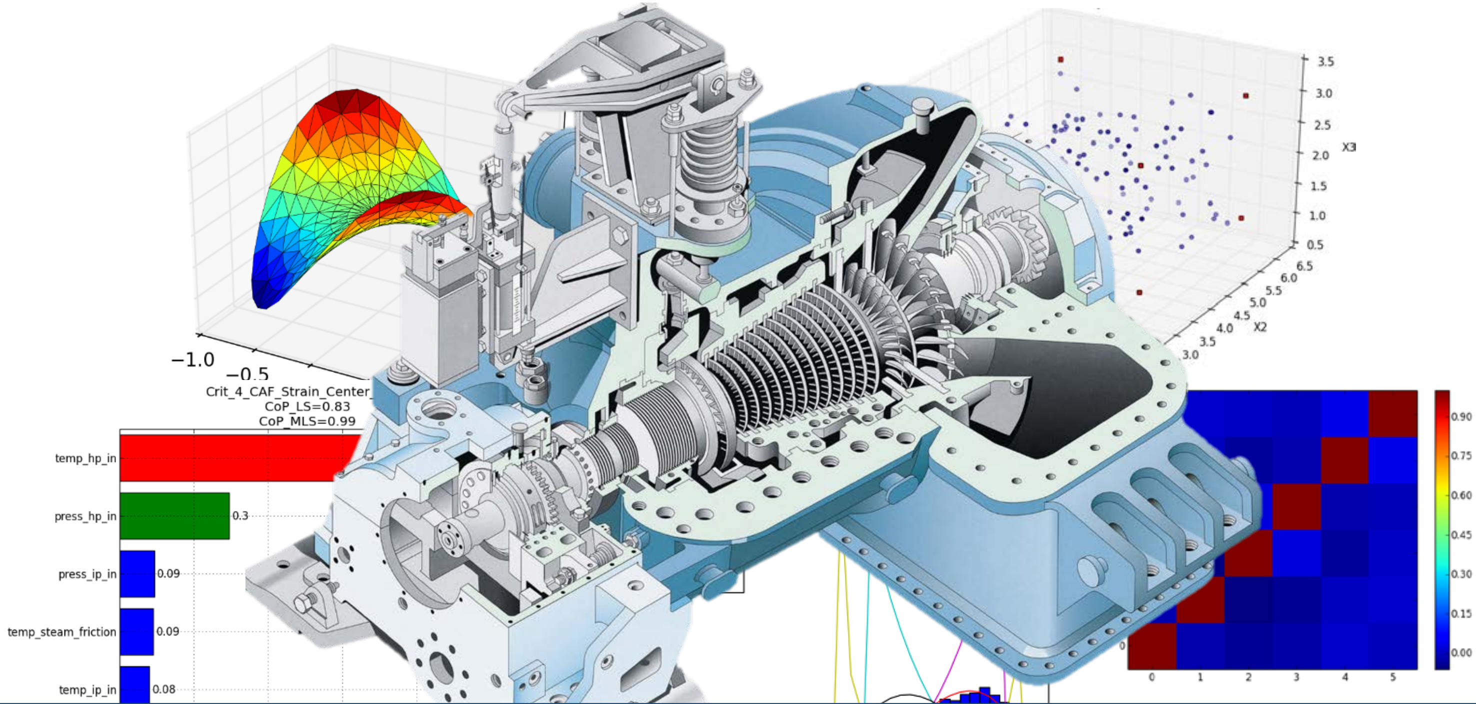

Requirements and new approaches of probabilistic optimal design from a practical point of view considering steam turbines

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Outline

1. Introduction
   - Motivation
   - Objective of the PhD thesis
   - Planned application

2. Previous results
   - Optimized latin hypercube sampling
   - New moving least square approximation

3. Outlook

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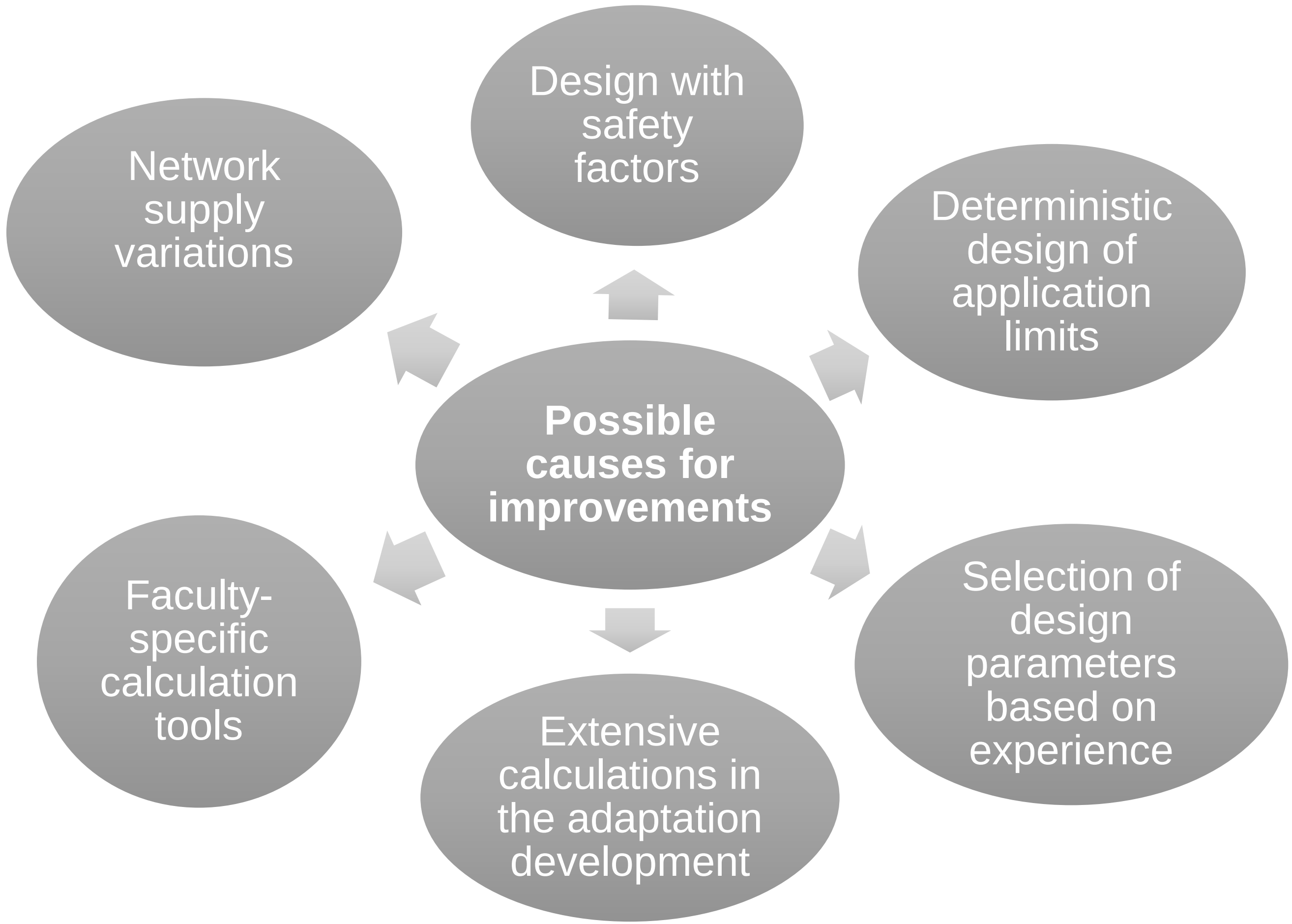
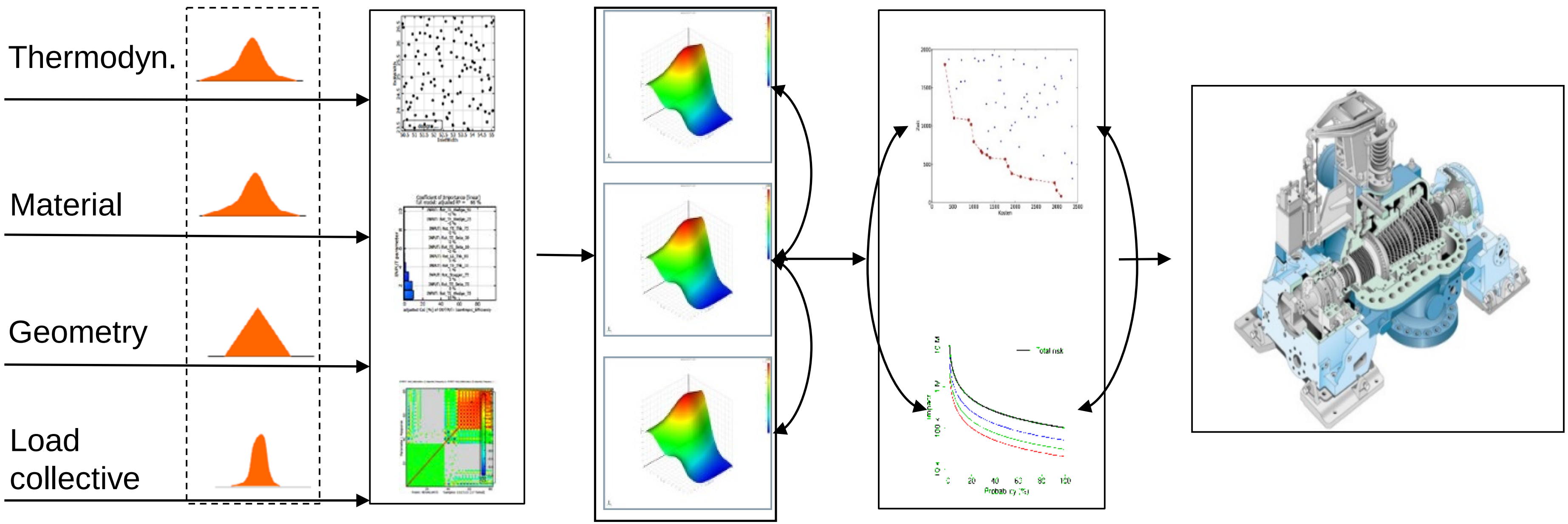
1. Introduction - Motivation



1. Introduction - Objective of the PhD thesis

Input parameters | **Design-sampling** | **Faculty-specific metamodels** | **Reliability-based multi-objective optimization** | **Optimal robust design taking into account the probability of failure**

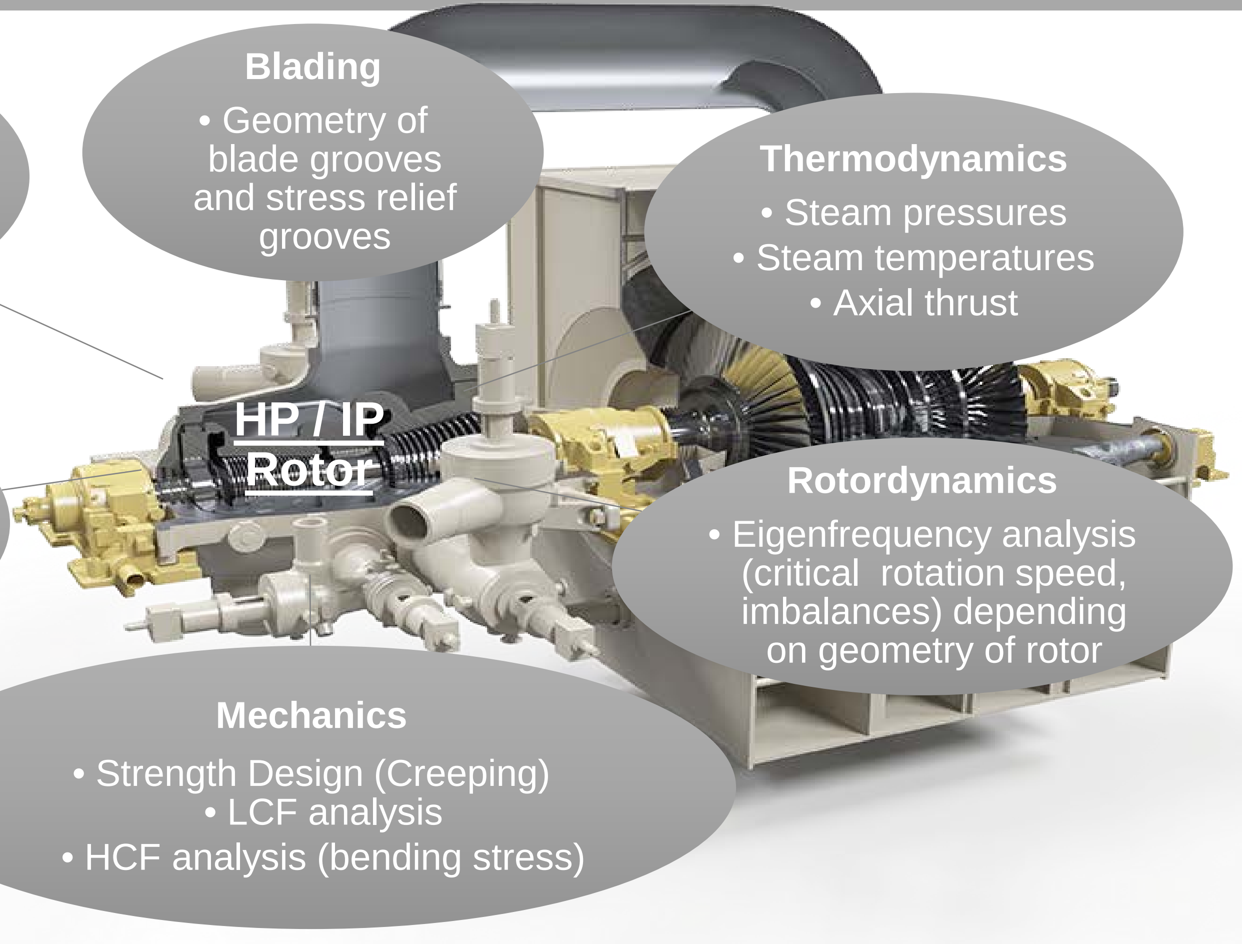


1. Introduction - Objective of the PhD thesis

The advantages:

- Failure probabilities instead of safety factors
- Better understanding of parameters through sensitivity analysis
- Increased flexibility to perform changes within the parameter space
- The possibility to use extended application limits in contrast to the deterministic design
- Avoid interface conflicts and no need for expert knowledge in all areas / tools
- Optimal compromise solutions for the requirements

1. Introduction – Planned application



Operations
- Start-up time
- Time for load changes
- Allowable part-load operation

Blading
- Geometry of blade grooves and stress relief grooves

Thermodynamics
- Steam pressures
- Steam temperatures
- Axial thrust

Fracture mechanics
- Crack growth

Rotordynamics
- Eigenfrequency analysis (critical rotation speed, imbalances) depending on geometry of rotor

Mechanics
- Strength Design (Creeping)
- LCF analysis
- HCF analysis (bending stress)

1. Introduction – Planned application

„External" constraints:

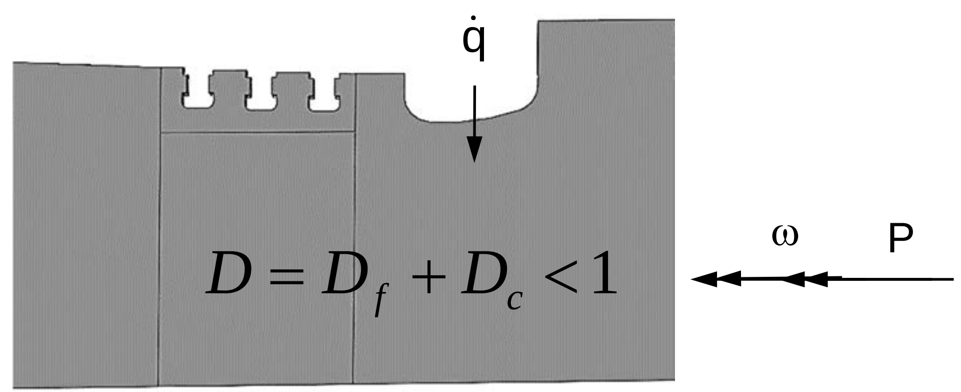
- Steam temperature (THD)
- Start times
- Required lifetime
- Number of starts
- Number of load cycles
- …

„Internal“ constraints :

- Fulfillment mechanical integrity rotor + blading
- Manufacturability
- Compliance criteria rotor dynamics
- …

Objectives:

- Long lifetime (h)
  - ➢ High number of starts (N)
  - ➢ High start speed (MW / min)
  - ➢ High number of load cycles (N)
- High performance (MW)
- High efficiency (mu)
- Eigenfrequencies outside critical regions (50Hz, 60 Hz, 100Hz, 120Hz)
-



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2. Previous results - Optimized latin hypercube sampling

Base : **standard latin hypercube sampling**

- Parameter space is divided into N (samples or classes) with the same probability of 1 / N, and in each of these classes a random point is selected.

Advantages:

➢ Low computation time to generate.

➢ Can be used with a large number of samples N and input parameters n.

➢ It has a lower variance compared to standard Monte Carlo sampling.

➢ The value range of each variable is “completely” covered.

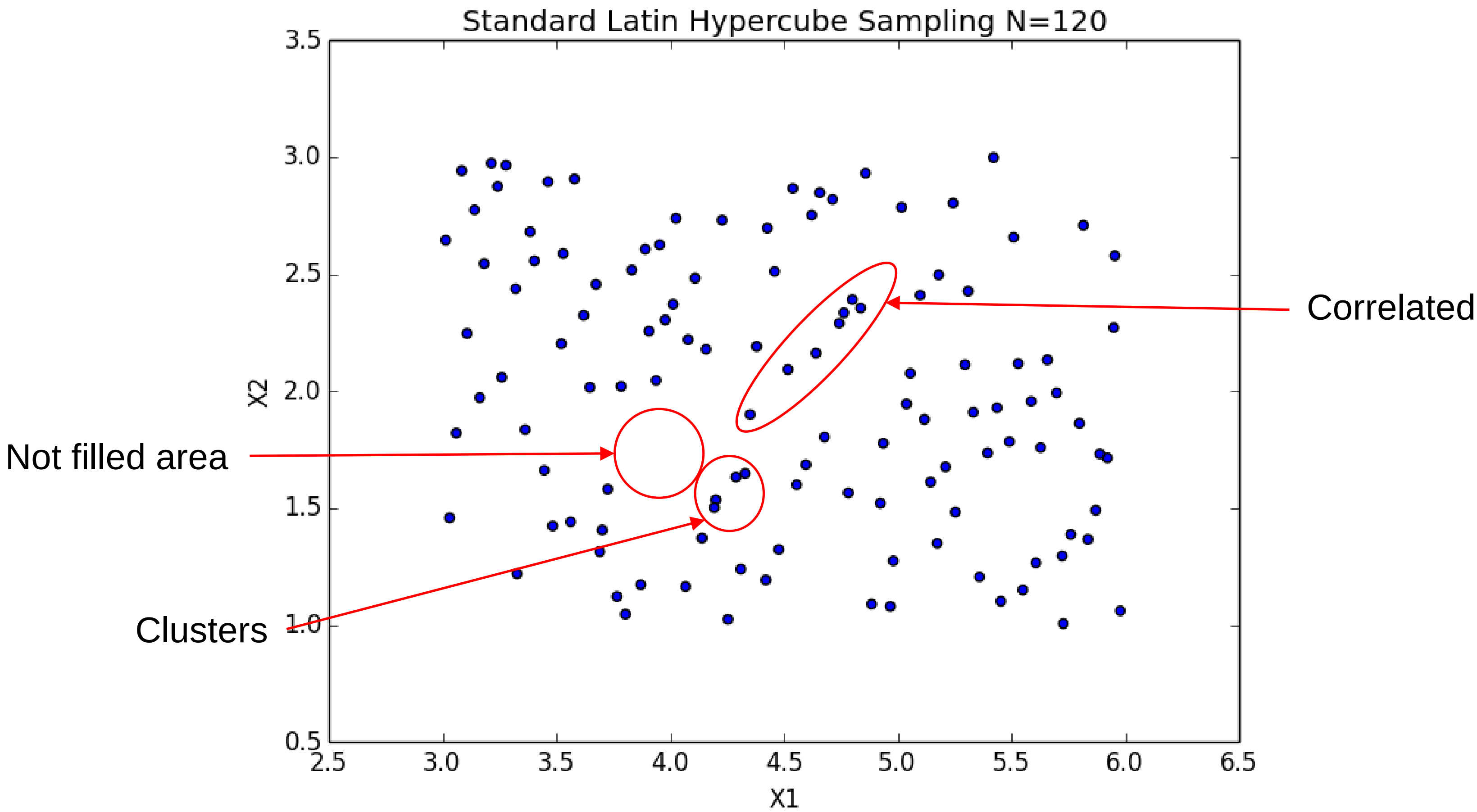
Cons:

➢ It can not be specified multivariate dependencies.

➢ It can cause unwanted input parameter correlations.

➢ It does not guarantee a "filling" coverage of the parameter space.

2. Previous results - Optimized latin hypercube sampling



2. Previous results - Optimized latin hypercube sampling

Optimized latin hypercube sampling:

- The standard latin hypercube sampling is improved by optimization

- Optimization criteria are:
  - Correlation coefficient by Owen [1]: $\rho^2 = \frac{\sum_{i=2}^{k} \sum_{j=1}^{i-1} \rho_{ij}^2}{k(k-1)/2}$; evaluation of the correlation.
  - Maximin design coefficient by Morris and Mitchell [2] : $\phi_p = (\sum_{i=1}^{m} J_i D_i^{-p})^{1/p}$; evaluation of the distance between the design points.

- Overall criterion: $f(\rho^2, \phi_p) = w_1 \rho^2 + w_2 \phi_p = \Psi$; with $w_1 + w_2 = 1$

- Optimization method: **Simulated Annealing (SA)** [3]

2. Previous results - Optimized latin hypercube sampling

Simulated Annealing (SA):

- Start is a random LHS with a design matrix D where each column stands for a design point.
- By interchanging p (p < = n) elements of two randomly selected columns -> D_{try} is a new design matrix .
- If $\psi(D_{try})$ is better than $\psi(D)$ D gets D_{try}. If $\psi(D_{try})$ is worse than $\psi(D)$ a random decision is made, if still D gets D_{try} or whether D_{try} is discarded. That D gets D_{try} will happen with the probability:

  $\pi = exp(-[\Psi(D_try) - \Psi(D)]/t)$, t=self-selected parameter.

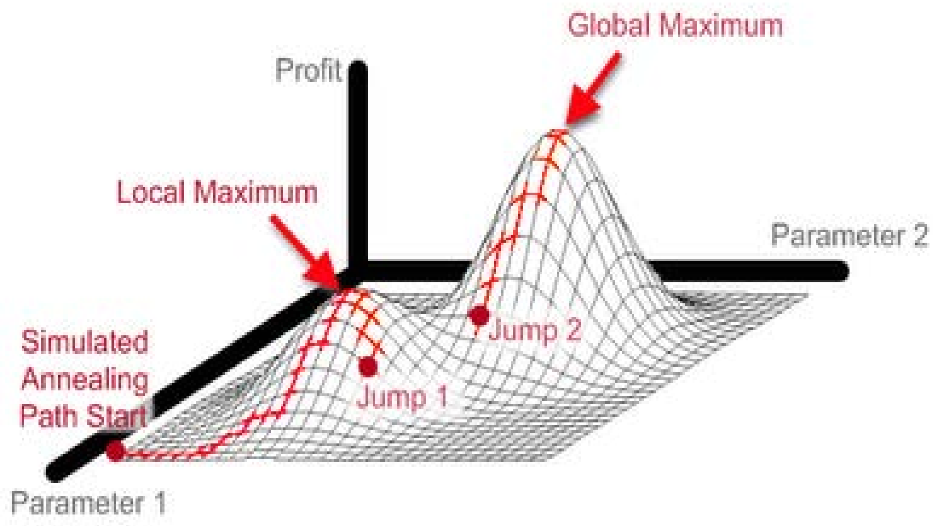
2. Previous results - Optimized latin hypercube sampling

- This random decision prevents that only a local minimum is found.

- The result of the optimization is D_{best}.

- Futher parameters of this algorithm :
  - FAC_t Factor to t is minimized e.g: $FAC_t = 0.95$.
  - I_{max} = Number of iterations before t is minimized to FAC_t.

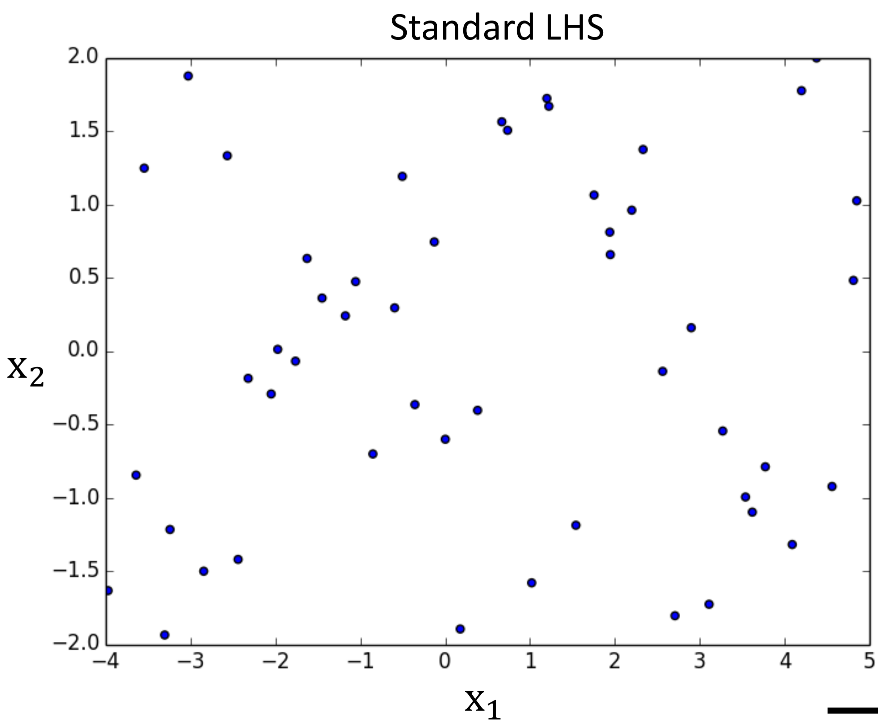
- Start Designmatrix D is the best sampling regarding the overall ciriterion ψ out of 500 generated standard latin hypercube samplings



(source: [10])

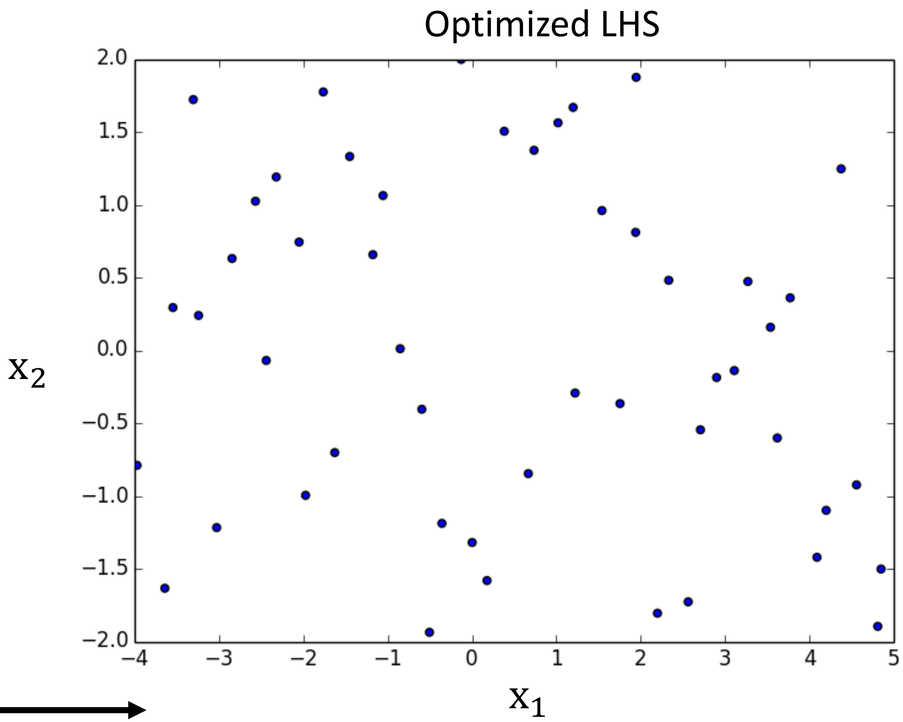
2. Previous results - Optimized latin hypercube sampling

Comparison for N=50 n=4:



$\psi = 0.246$

after optimization



$\psi = 0.050$

2. Previous results - New moving least square approximation

Standard moving least square approximation [4] :

$\widetilde{y}(x) = p^T(x)a(x)$; Approximation of the results of the test points

$p(x) = [1, x_1, x_2, ..., x_n, x_1^2, x_2^2, ..., x_n^2, x_1 x_2, ...x_n x_m]^T$; Polynomial basisfunction

$a(x) = (P^T W(x) P)^{-1} P^T W(x) y(x)$; Moving coefficients depending on testpoint x

$P = [p^T(x_1) p^T(x_2) ... p^T(x_N)]$; Contains all polynomial basisfunction of the supportpoints

$y(x) = [y(x_1) y(x_2) ... y(x_N)]$; Contains the results of the objective function for all support points

$W(x) = diag[w_1(x - x_1) w_2(x - x_2) ... w_N(x - x_N)]$; Overall weighting matrix (diagonal matrix) contains for each testpoint a seperate weighting

$$w_N(\|x - x_N\|) = \begin{cases} e^{-(\frac{\|x - x_N\|}{D\alpha})^2} & \|x - x_N\| \leq D \\ 0 & \|x - x_N\| > D \end{cases}$$; Gaussian weighting function

$$\alpha = \frac{1}{\sqrt{-log_{10}(0.001)}}$$; Constant

$D =$ Self- selected parameter affecting model accuracy

2. Previous results - New moving least square approximation

New moving least square approximation:

- A new concept of weighting, which not only includes a weighting for each test point, but also per variable . This means there exist not only one weighting matrix and D for the approximation but a weighting matrix and D per variable and if present for the crossterms.
- The D's are chosen through optimization with a particle swarm optimization algorithm.
- The optimization objective is the generalized coefficient of determination:

$$R^2 = \left(\frac{\sum_{N=1}^{k}(y^k - \mu_y)(\hat{y}^k - \mu_{\hat{y}})}{(N-1)\sigma_y \sigma_{\hat{y}}}\right)^2; 0 \leq R^2 \leq 1$$

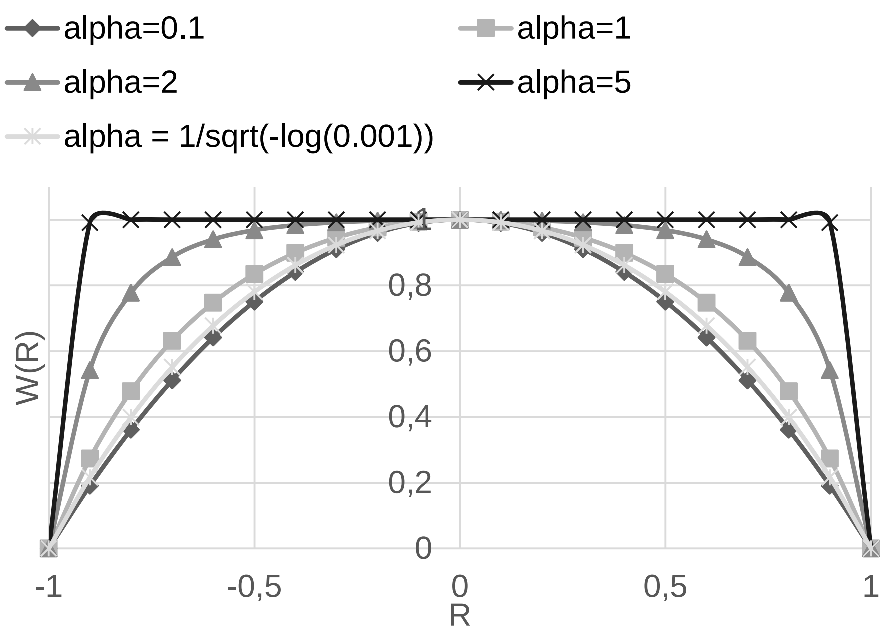
- Whereby the sample points are divided in equal subsets (0.2*N), so that every sample point is support- and testpoint, then the average R^2 is calculated (cross validation).
- α is not longer a constant, but a further optimization variable, in order to better fit the problem, rather than to change the weighting function. Therefore is also used a new formulation of the Gaussian weighting function:

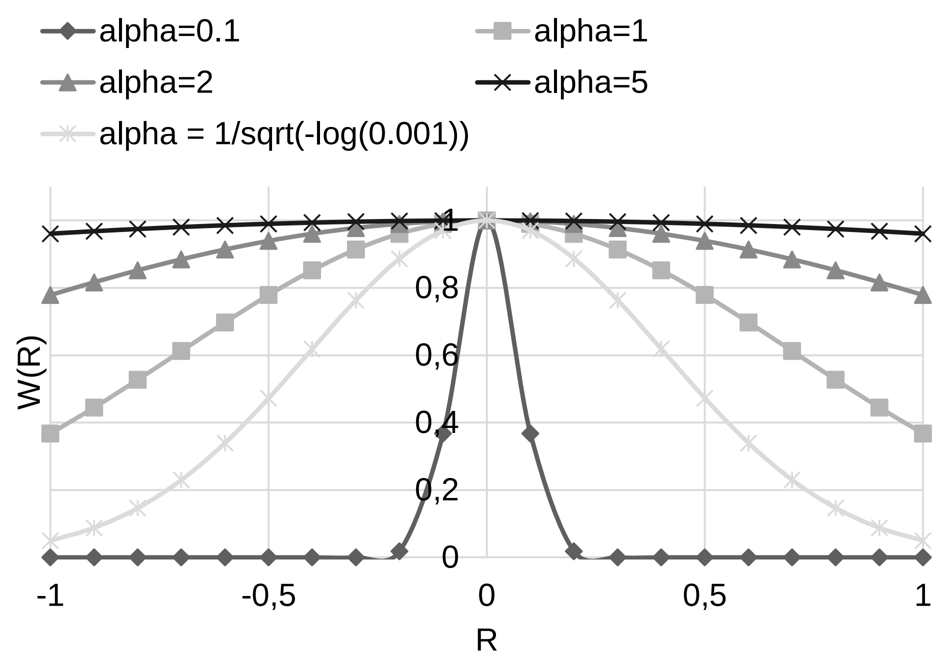
$$w_N(\|x - x_N\|) = \begin{cases} \frac{e^{-(\alpha\|x-x_N\|/D)^2} - e^{-\alpha^2}}{(1-e^{-\alpha^2})} \\ 0, \end{cases}$$

2. Previous results - New moving least square approximation

$$w_N(\|x-x_N\|) = \begin{cases} \frac{e^{-(\alpha\|x-x_N\|/D)^2} - e^{-\alpha^2}}{(1-e^{-\alpha^2})} \\ 0, \end{cases}$$

$$w_N(\|x-x_N\|) = \begin{cases} e^{-\left(\frac{\|x-x_N\|}{D\alpha}\right)^2} \\ 0 \end{cases}$$





Shape of the Gaussian weight functions with different control parameter α with $R = \|x - x_N\| / D$

2. Previous results - New moving least square approximation

Particle Swarm algorithm [5]:

- An initial population ("swarm") of possible candidates (“particle") move through the parameter space .
- The direction of the "particle" is guided by the knowledge of its best position (local optimum), and the best known position of the “swarm leader“ (global optimium).
- If new better positions are discovered, they are used to steer the “swarm".
- This process is repeated a certain number of iterations until an optimal solution is found.

Advantages:

- Suitable for multiobjective optimization.
- High number of input variables possible.

Disadvantage:

- Suitable coefficients for the “swarm" to determine behavior (8 coefficients).

(source: [6])

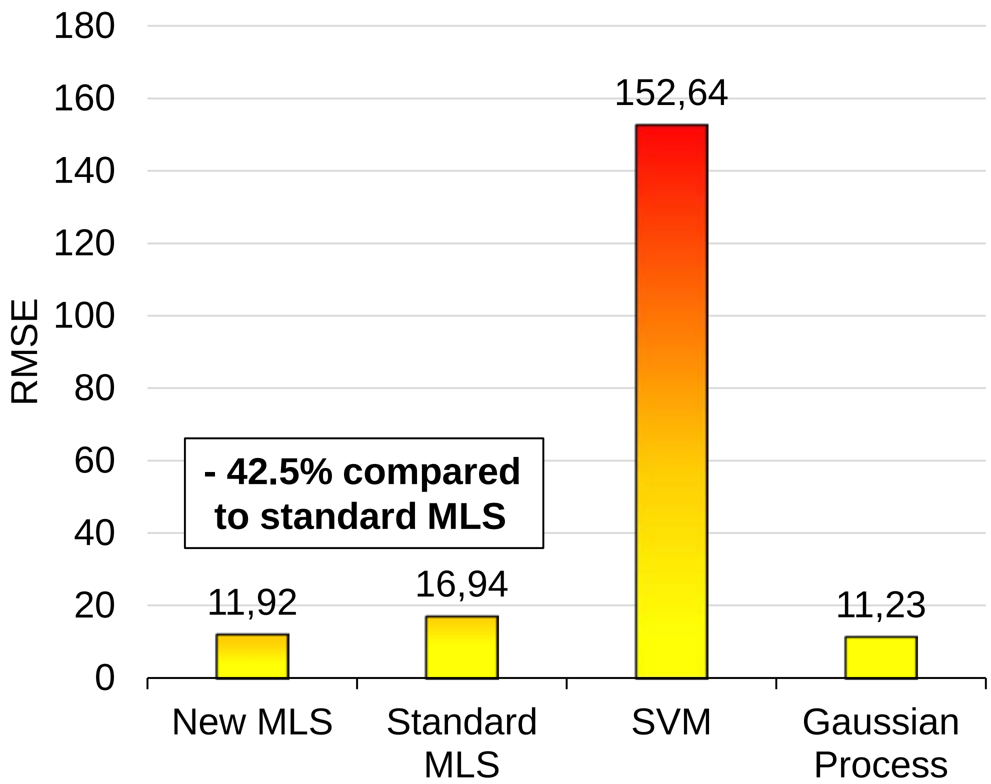
2. Previous results - New moving least square approximation

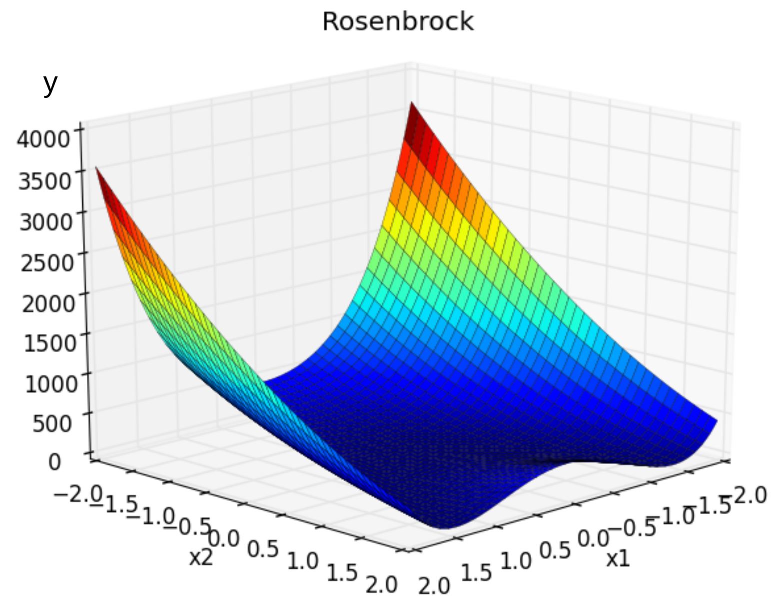
Benchmark of the new MLS:

- Used Metamodels [7]: standard MLS (D is also found through optimization), new MLS, super vector machine [8], Gaussian-process-regression [9](Kriging is a modification of this method).
- Evaluation criterion: $RMSE = \sqrt{\frac{1}{N_{test}} \sum_{k=1}^{N_{test}} (y(x_k) - \hat{y}_e(x_k))^2}$
- $N_{Support} = 120;\ N_{test} = 500; n = 2 - 4$
- Testfunctions:
  - Rosenbrock: $y = 100(x_1^2 - x_2)^2 + (x_1 - 1)^2; -2 \leq x_i \leq 2$
  - Normal PDF shape: $y = \frac{1}{1+x_1^4+5x_2^4+2x_2^2}; -3 \leq x_i \leq 3$
  - Sixhump Camelback: $y = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4; -2 \leq x_i \leq 2$
  - Rastrigin $y = x_1^2 + x_2^2 - cos(18x_1) - cos(18x_2); -1 \leq x_i \leq 1$
  - Own testfunction 1 $y = x_1 2 + x_2^2 + sin(x_3) + cos(x_4) + x_5^3; -5 \leq x_i \leq 5$
  - Own testfunction 2 $y = x_1 + x_1^2x_2^2 + sin(x_3)^2 + cos(x_4x_2) + x_3^4x_1; -5 \leq x_i \leq 5$

2. Previous results - New moving least square approximation

Rosenbrock: $y = 100(x_1^2 - x_2)^2 + (x_1 - 1)^2; -2 \leq x_i \leq 2$





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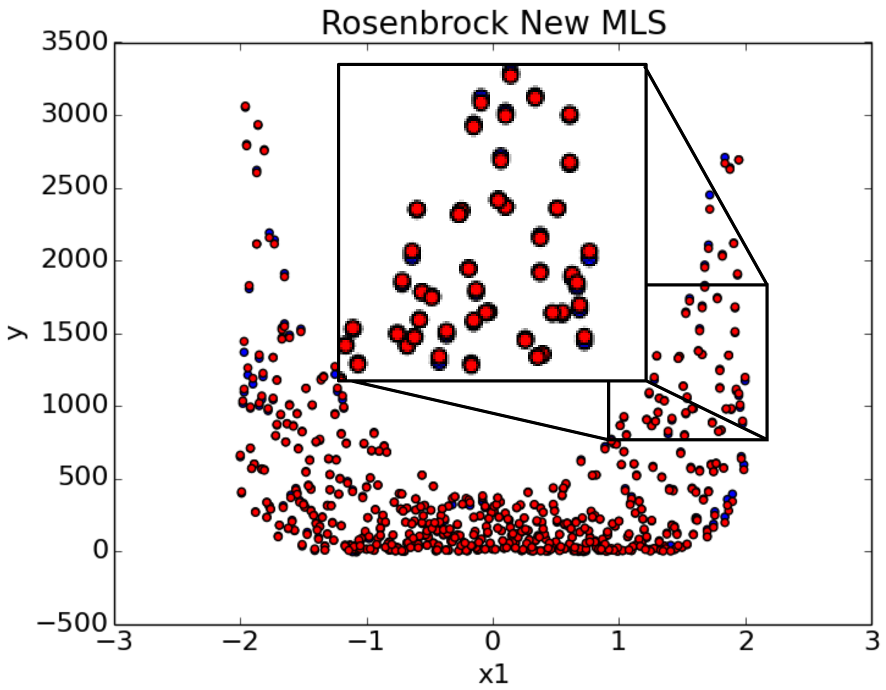
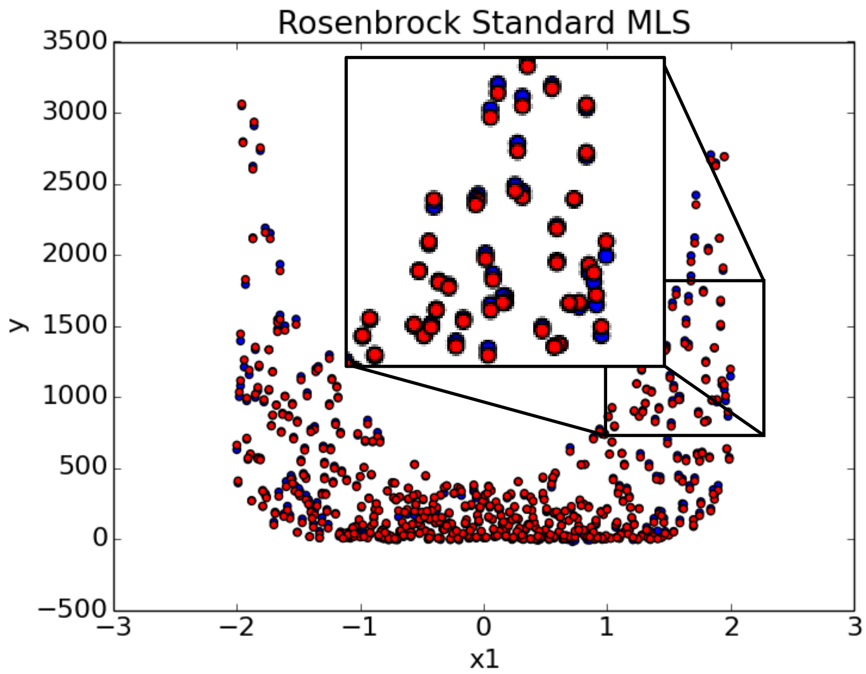


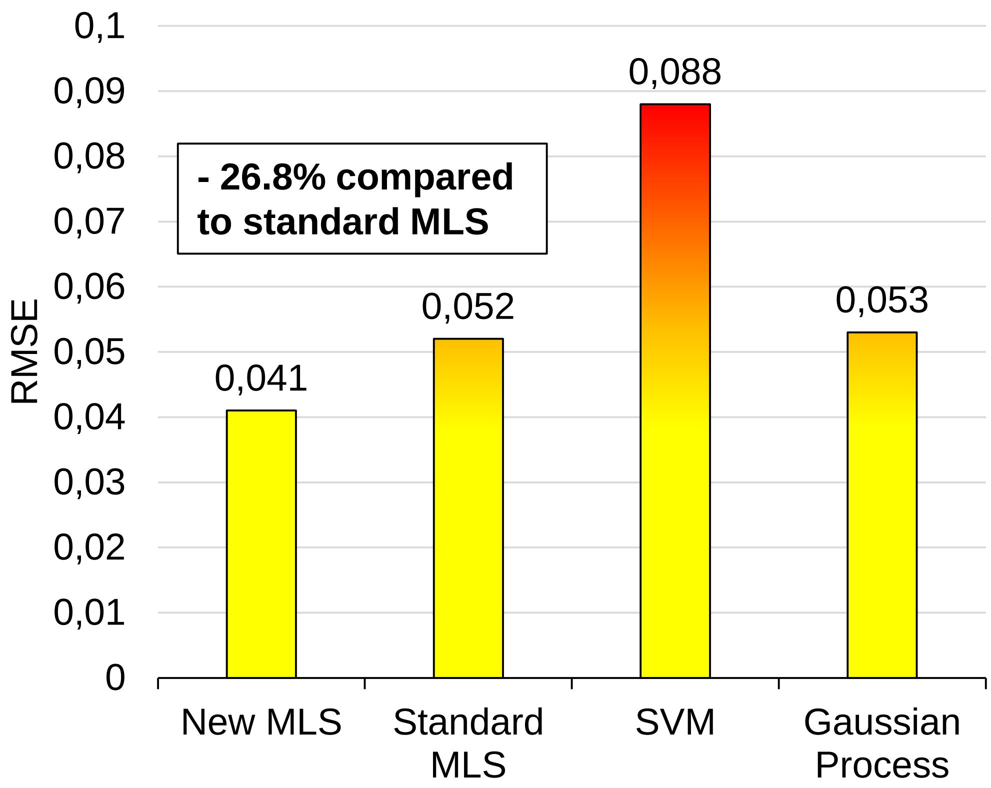


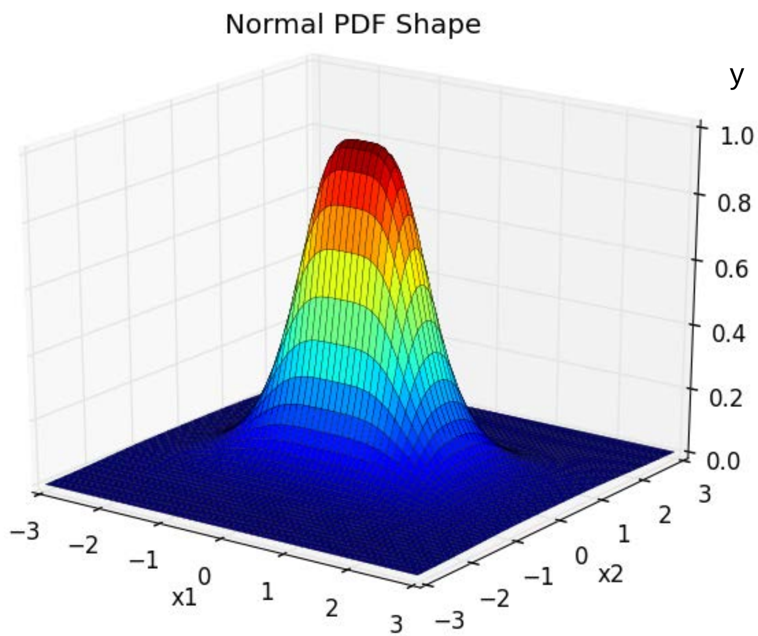
Blue = Approximation

Red = Calculation

2. Previous results - New moving least square approximation

Normal PDF shape: $y = \frac{1}{1+x_1^4+5x_2^4+2x_2^2}; -3 \leq x_i \leq 3$





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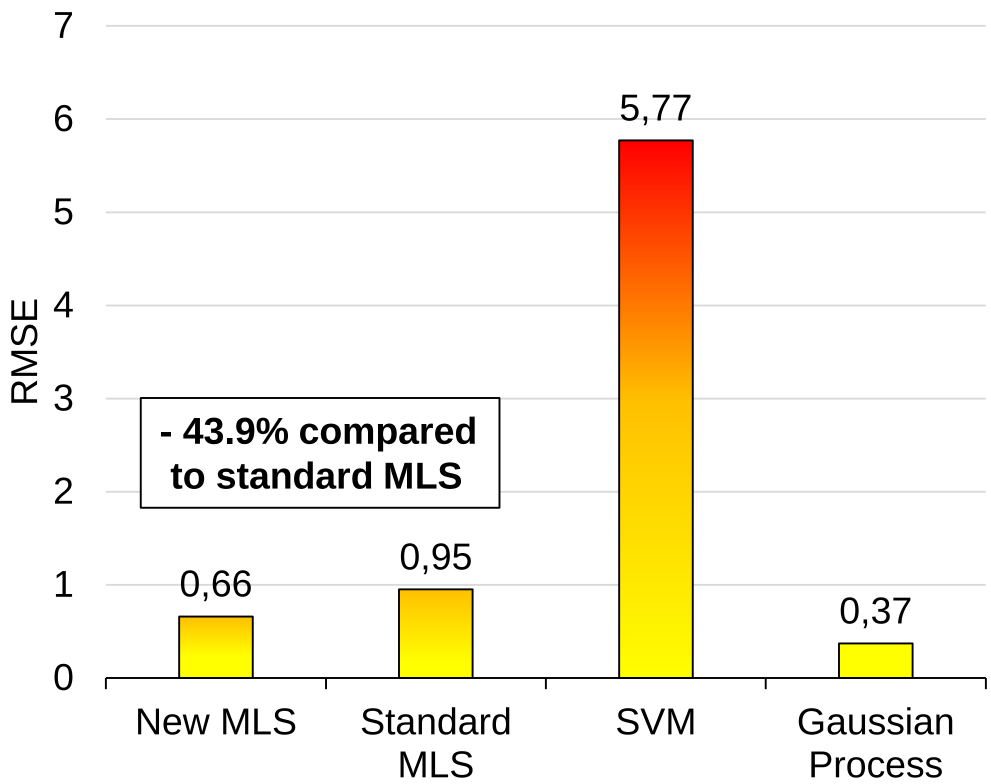
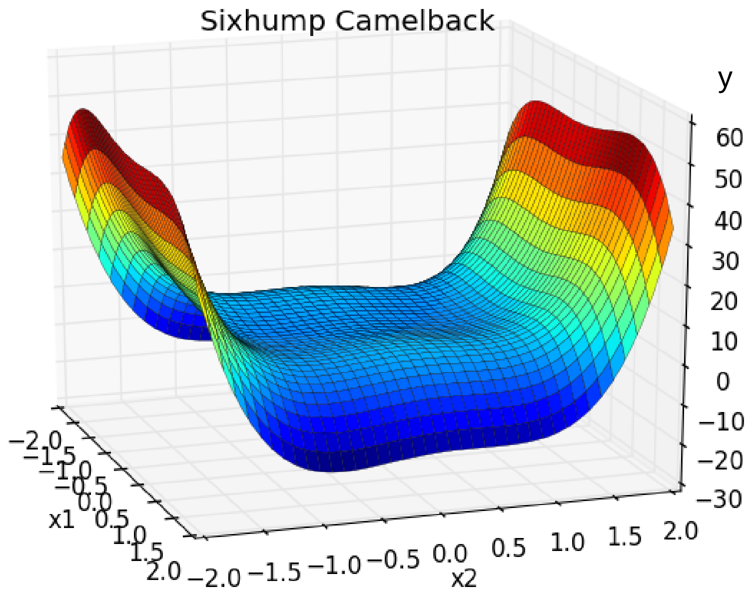
Sixhump Camelback: $y = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4; -2 \leq x_i \leq 2$





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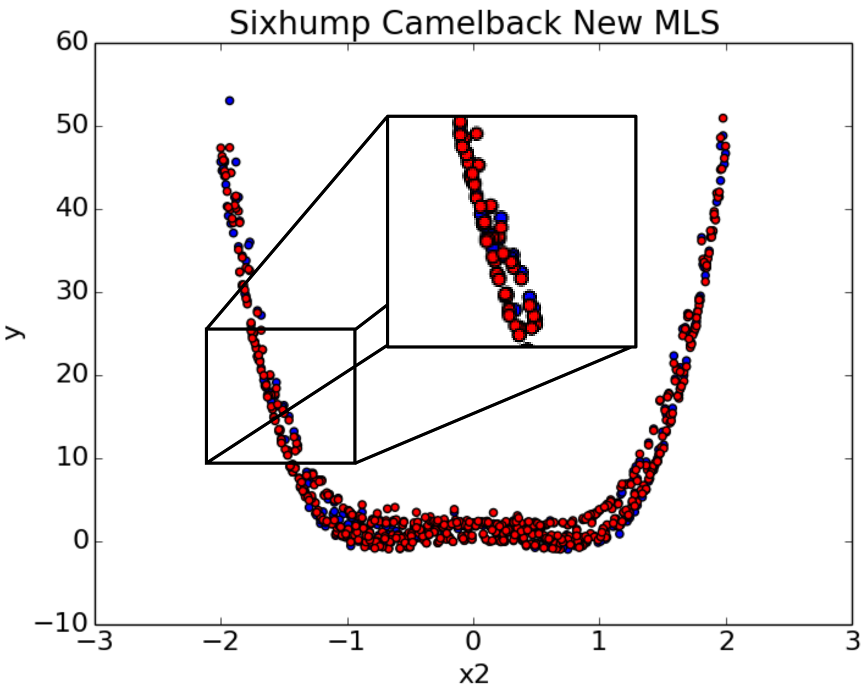

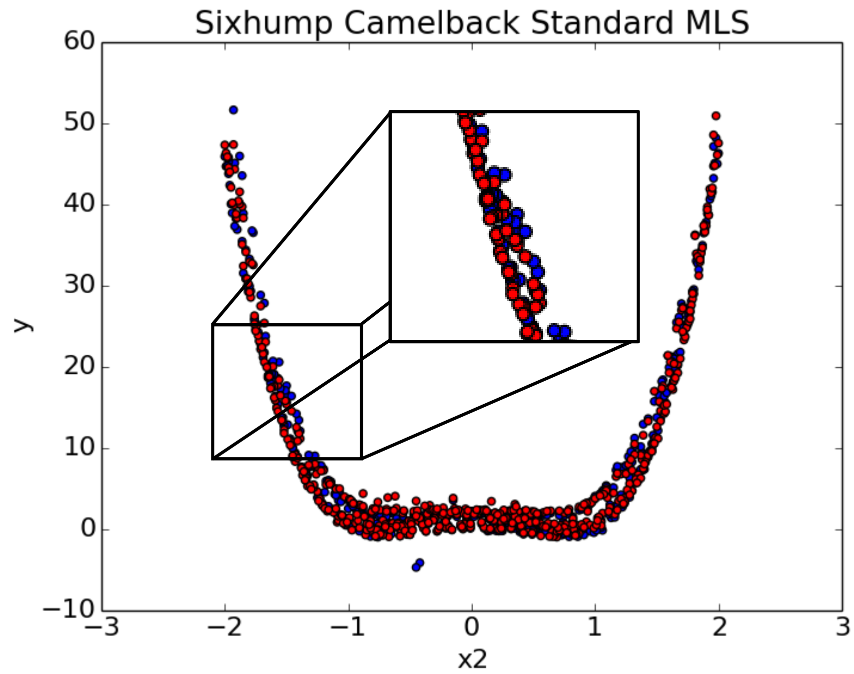

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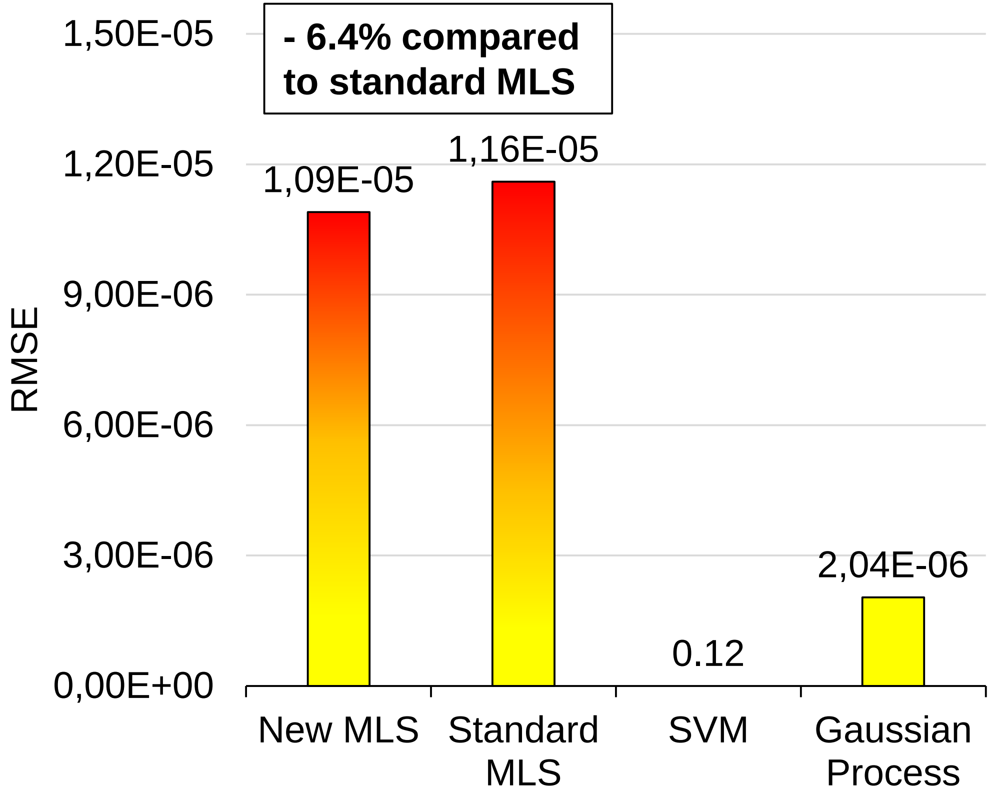
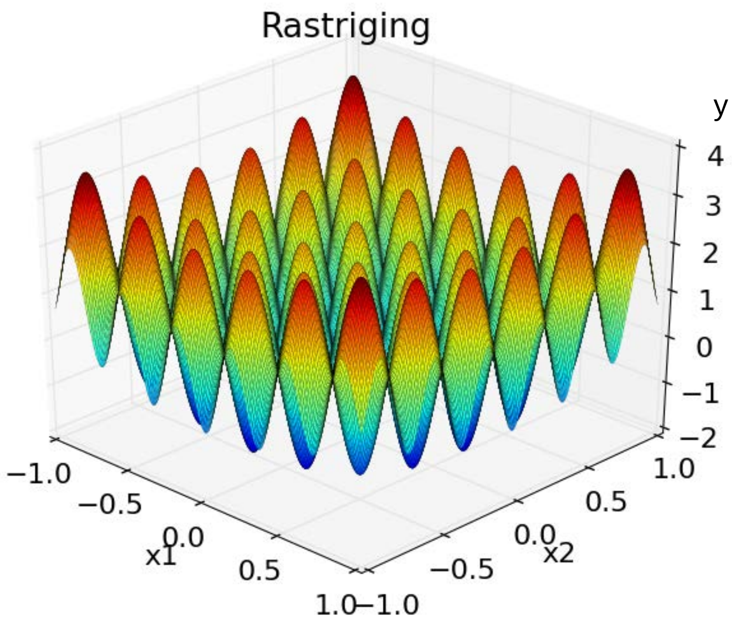
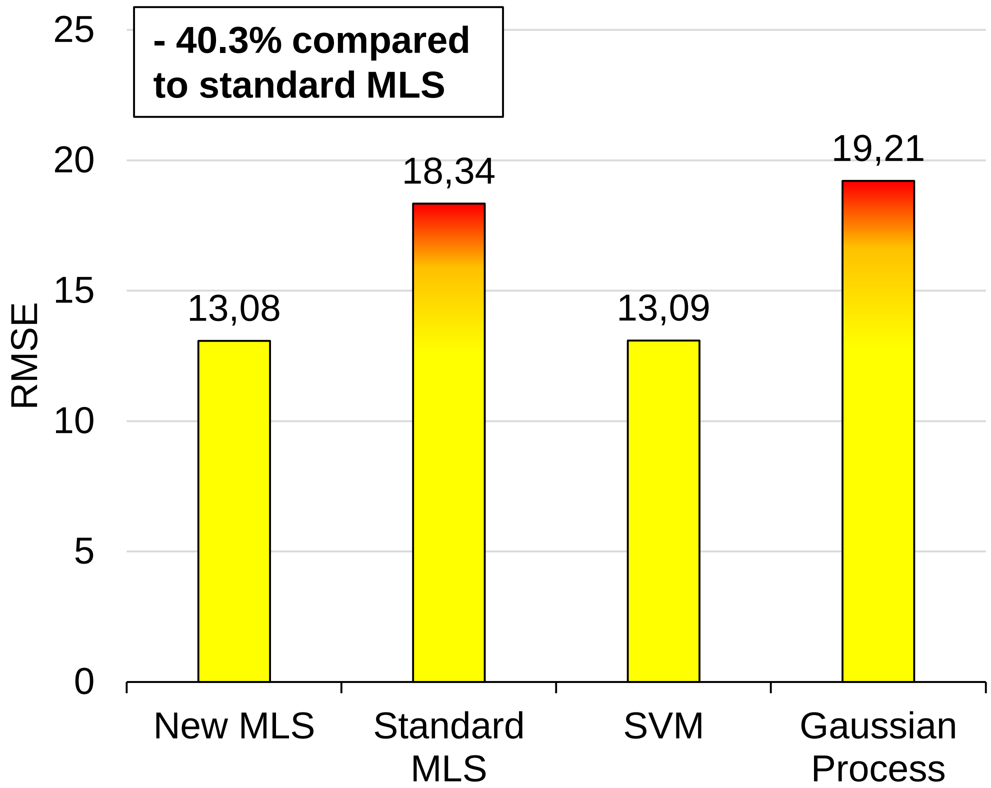
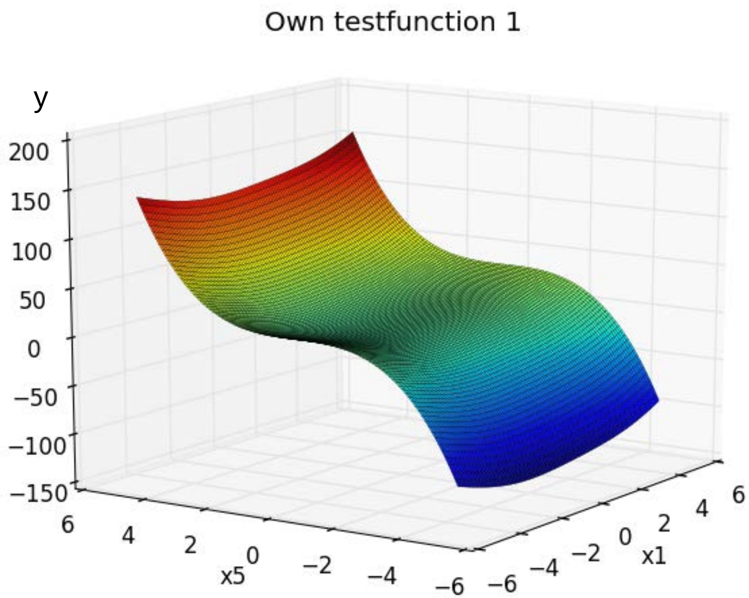




2. Previous results - New moving least square approximation

Own testfunction 1: $y = x_1 2 + x_2^2 + sin(x_3) + cos(x_4) + x_5^3; -5 \leq x_i \leq 5$





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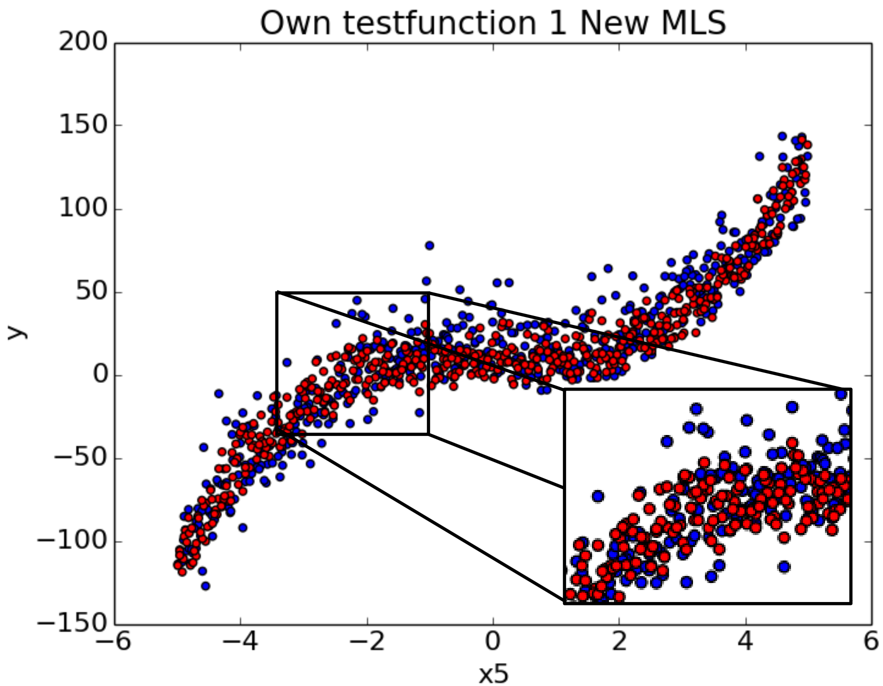

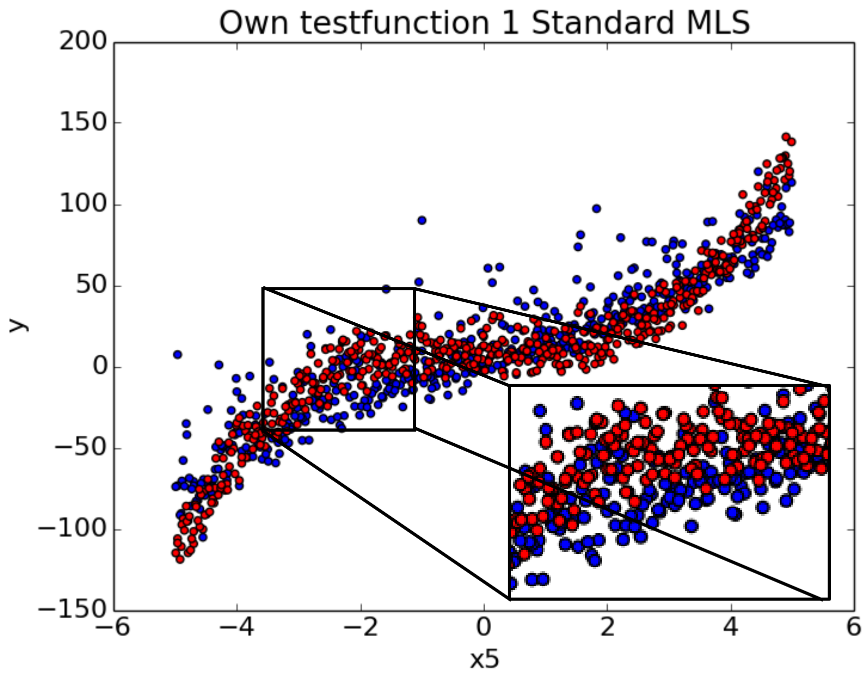

Blue = Approximation
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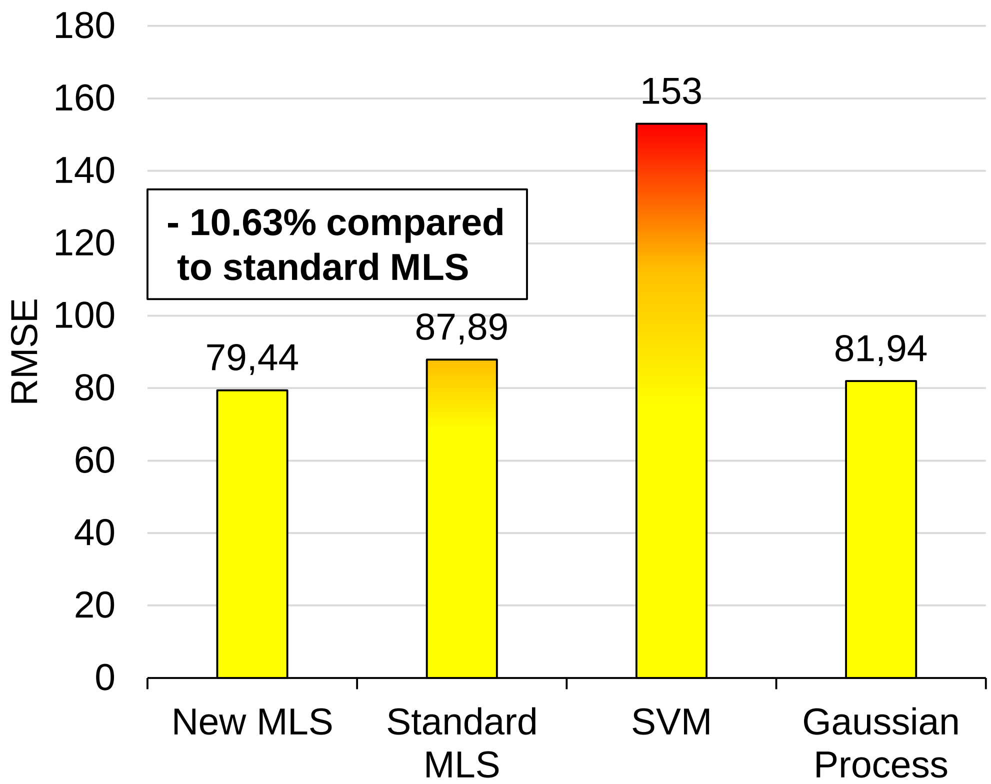
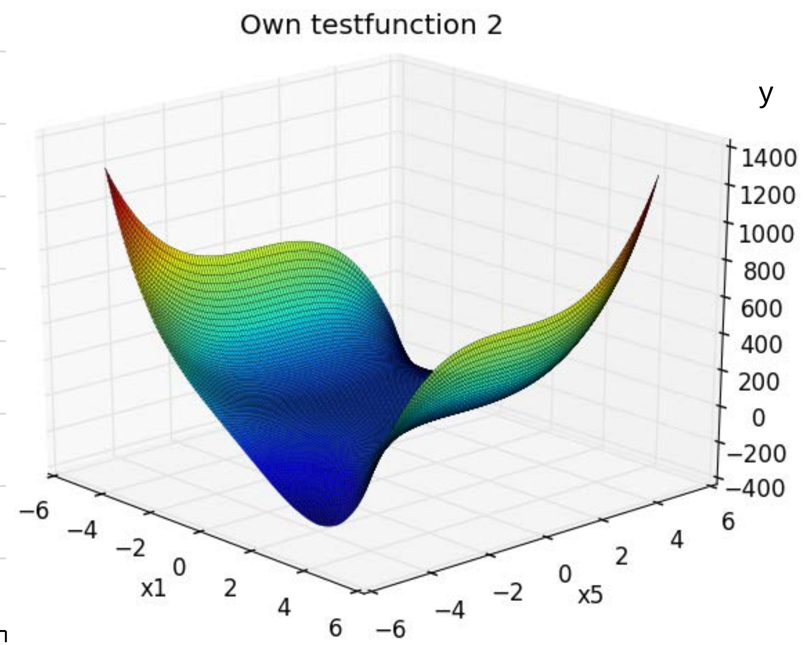
2. Previous results - New moving least square approximation

Own testfunction 2: $y = x_1 + x_1^2 x_2^2 + sin(x_3)^2 + cos(x_4 x_2) + x_3^4 x_1; -5 \leq x_i \leq 5$





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3. **Outlook**

3. Outlook

Developement of:

- New moving least square method (tests of different optimization algorithms for a high number of variables)
- Sampling method for distributed parameters for reliability analysis.
- Combination of sampling and estimination of the prognosis quality of the metamodell -> convergence analysis of the prognosis quality to sample the minimum number of designs.
- Chance-constraints stochastic multi objective optimization (optimization taking into account the probability of failure of the constraints).
- Simulationmodel of the hp/ip rotor.

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