Robust Design Optimization strategies for virtual prototyping

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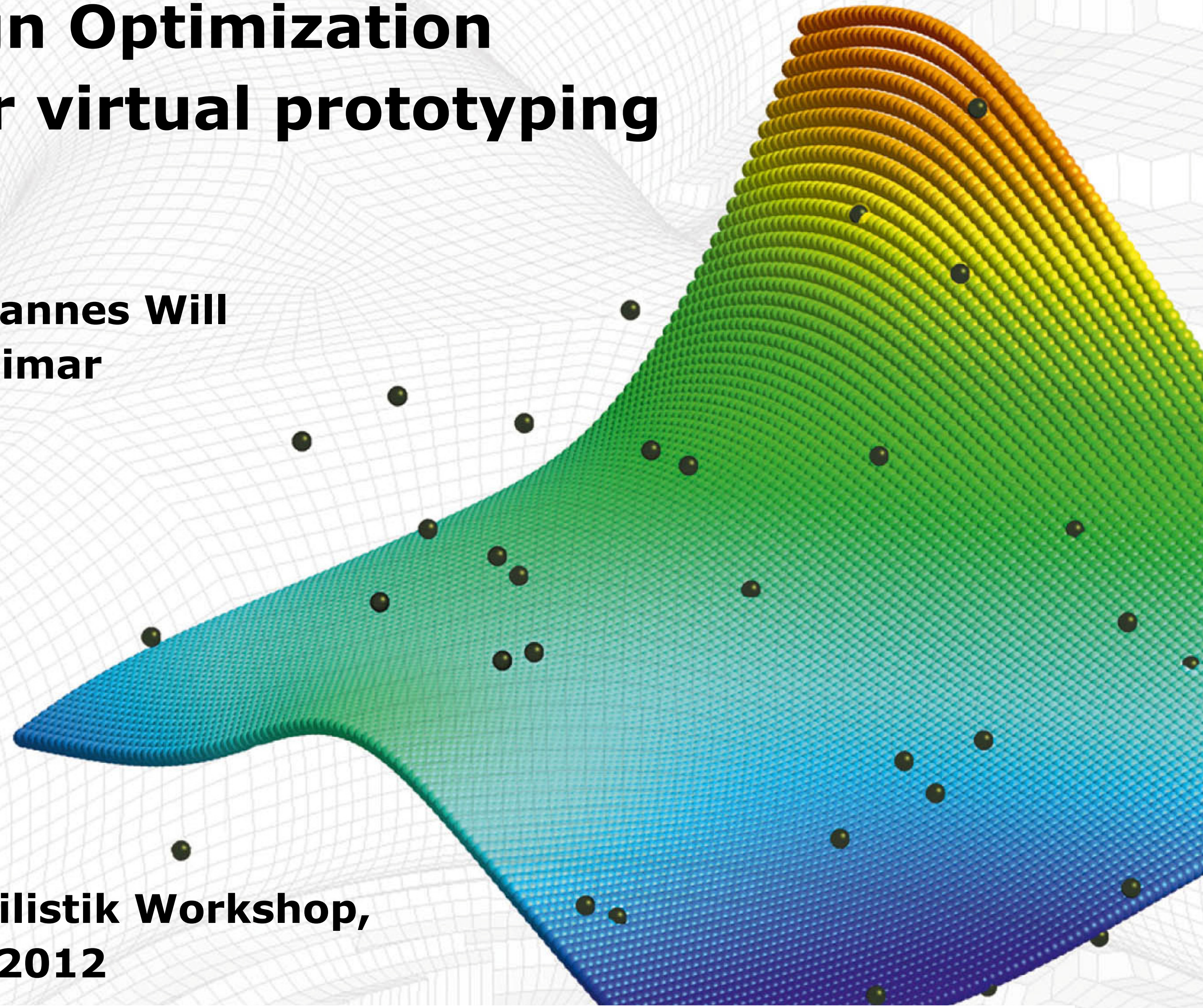
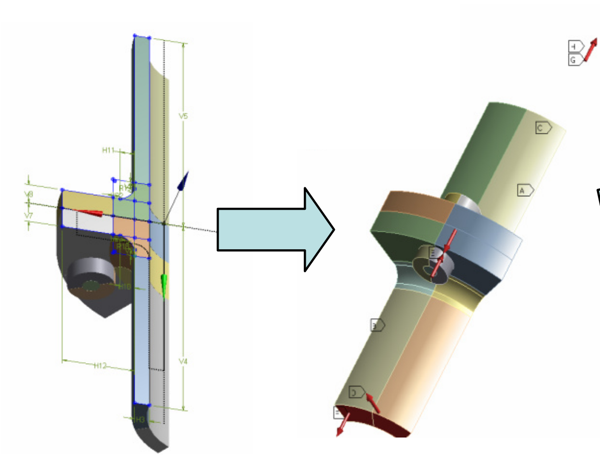


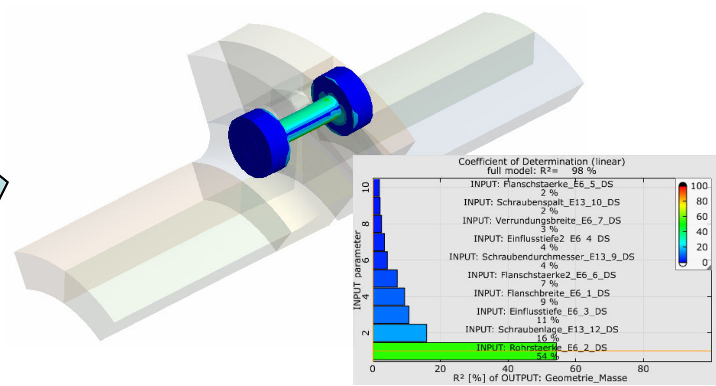
5. Dresdner Probabilistik Workshop,
27.-28. September 2012

Multidisciplinary Optimization

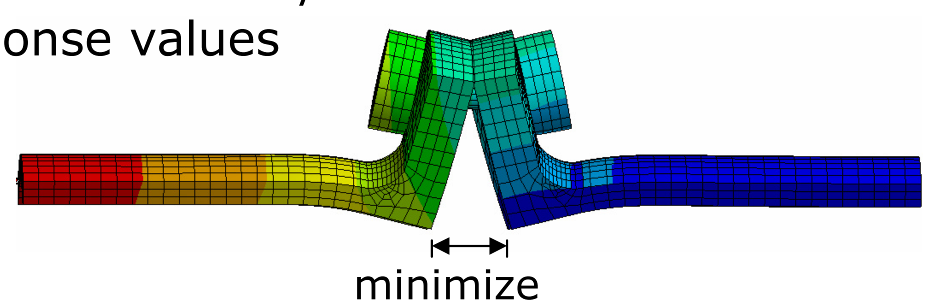
Multidisciplinary Optimization with optiSLang



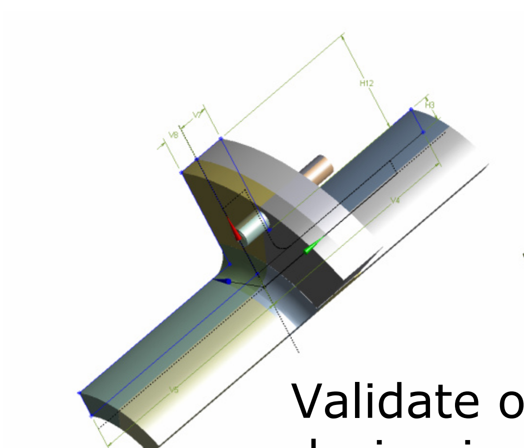
CAD and CAE Parameter definition

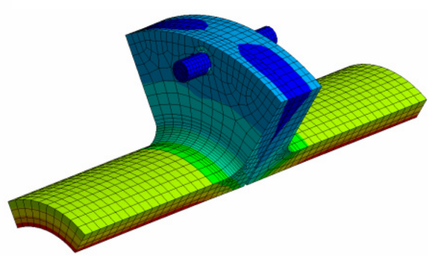


Sensitivity study – identify the most important parameters and check variation/COD of response values



Define optimization goal and optimize

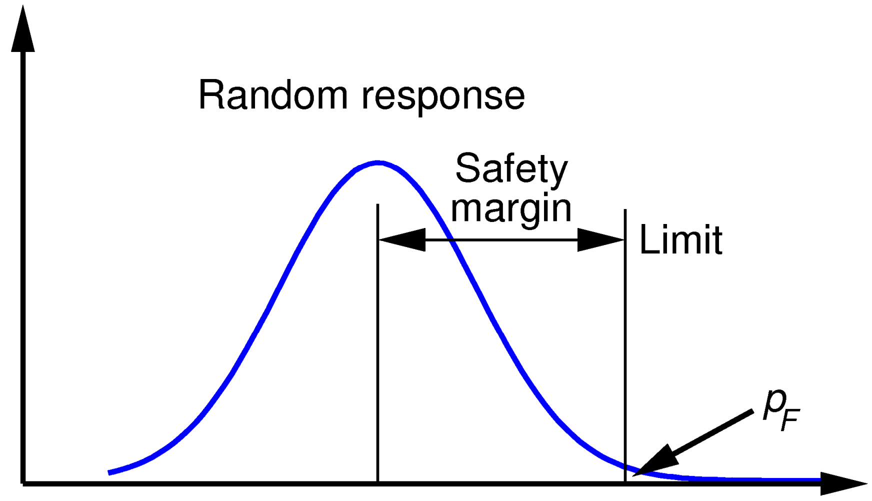




Validate optimized design in CAE and CAD

Robustness Analysis

Robustness in terms of constraints



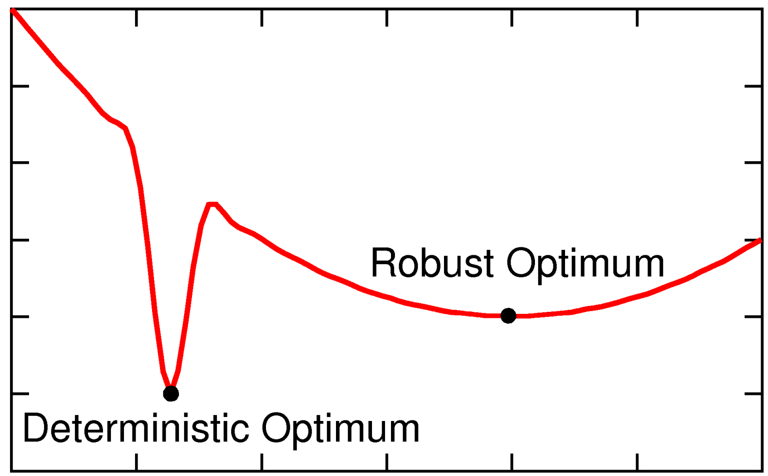
- Safety margin (sigma level) of one or more responses *y*:

$$y_{limit} - y_{mean} \leq a \cdot \sigma_y$$

- Reliability (failure probability) with respect to given limit state:

$$p_F \leq p_F^{target}$$

Robustness in terms of the objective



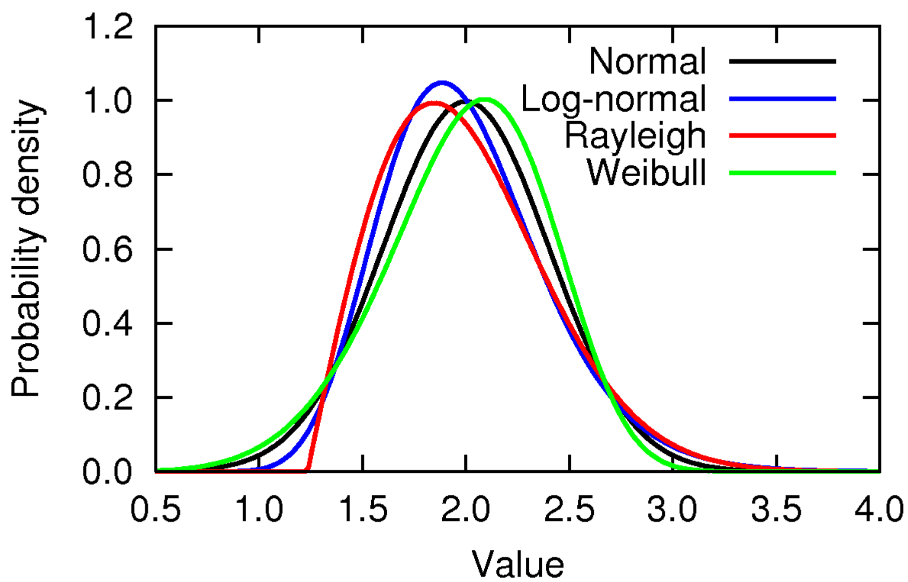
- Performance (objective) of robust optimum is less sensitive to input uncertainties
- Minimization of statistical evaluation of objective function *f* (e.g. minimize mean and/or standard deviation):

$$\bar{f} \rightarrow min \text{ or } \bar{f} + \sigma_f \rightarrow min$$

Sigma level vs. failure probability

- The sigma level can be used to calculate the probability of exceeding a certain response value
- Since the distribution type is often unknown, this estimate may be very inaccurate for small probabilities
- The sigma level deals with single limit values, whereas the failure probability quantifies the event, that any of several limits is exceeded

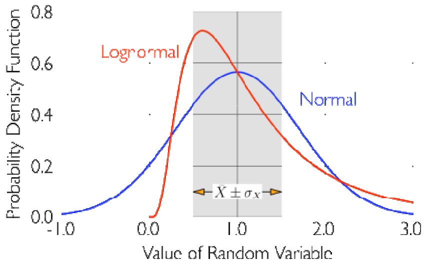
➢ Reliability analysis should be applied to proof the required safety level

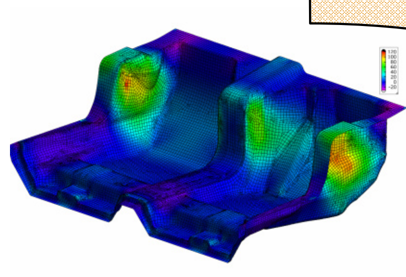


Distribution	Required sigma level (CV=20%)		
	$p_F = 10^{-2}$	$p_F = 10^{-3}$	$p_F = 10^{-6}$
Normal	2.32	3.09	4.75
Log-normal	2.77	4.04	7.57
Rayleigh	2.72	3.76	6.11
Weibull	2.03	2.54	3.49

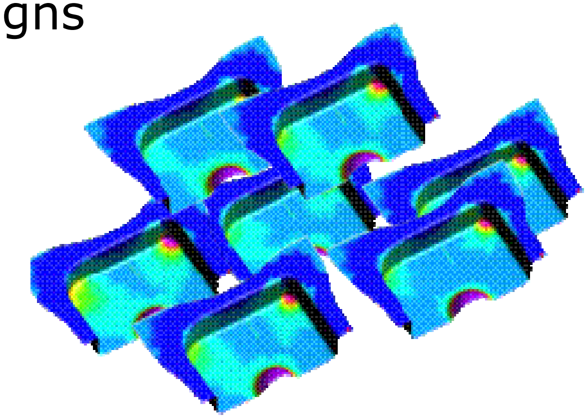
Variance based robustness analysis

1) Define the robustness space using scatter range, distribution and correlation

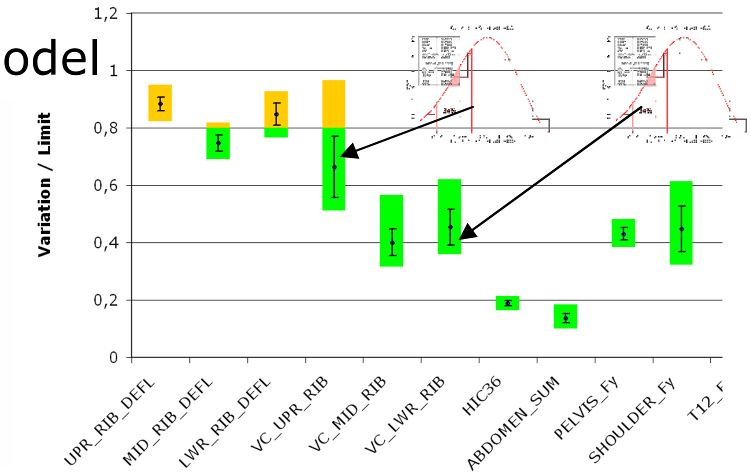




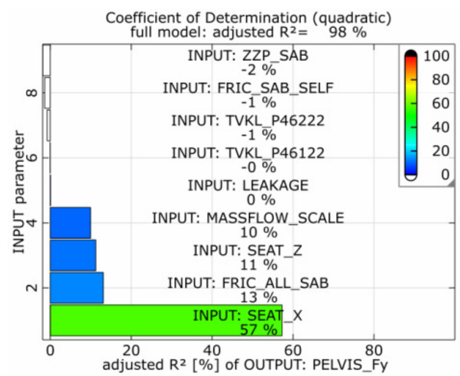
2) Scan the robustness space by producing and evaluating *n* designs



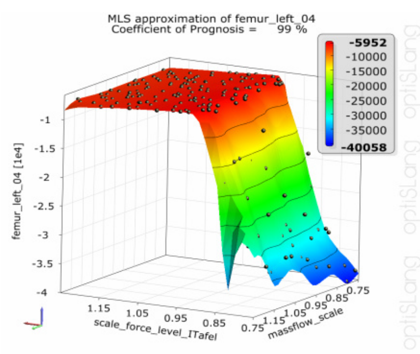
3) Check the variation



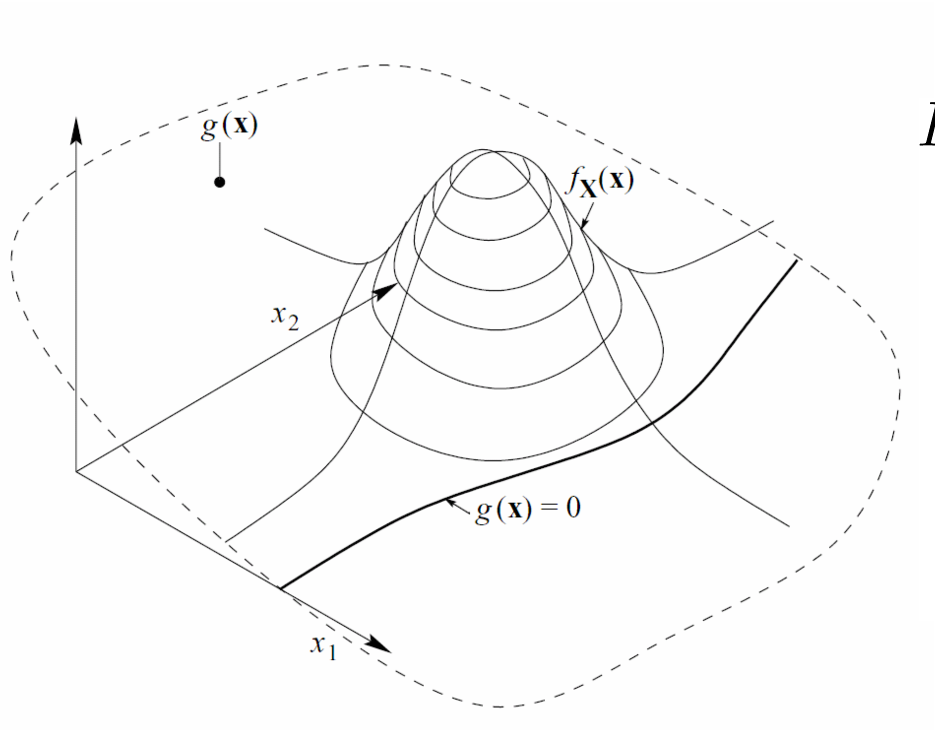
4) Check the explainability of the model



5) Identify the most important scattering variables



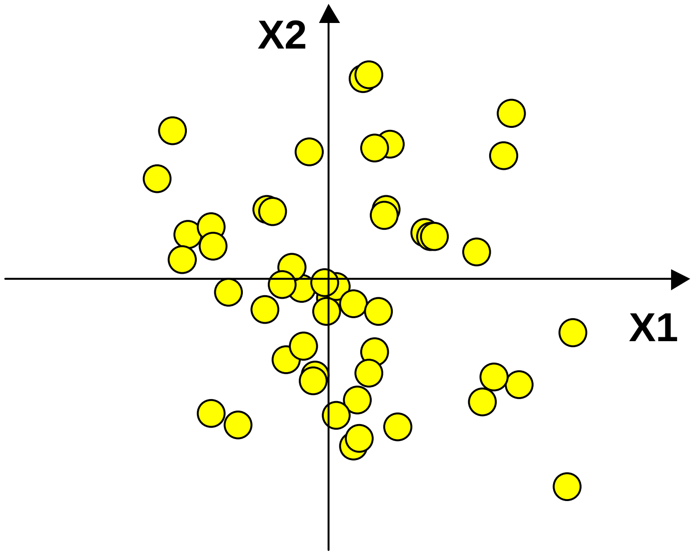
Reliability analysis



$$P_F = P\left[\mathbf{X} : g(\mathbf{X}) \leq 0\right]$$
$$= \int \cdots \int_{g(\mathbf{x}) \leq 0} f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$
$$= \int_{-\infty}^{\infty} \cdots \int_{-\infty}^{\infty} I\left(g(\mathbf{x})\right) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$$

- Limit state function *g(**x**)* divides random variable space **X** in safe domain *g(**x**)*>0 and failure domain *g(**x**)* ≤0
- Multiple failure criteria (limit state functions) are possible
- Failure probability is the probability that at least one failure criteria is violated (at least one limit state function is negative)
- Integration of joint probability density function over failure domain

Monte Carlo Simulation



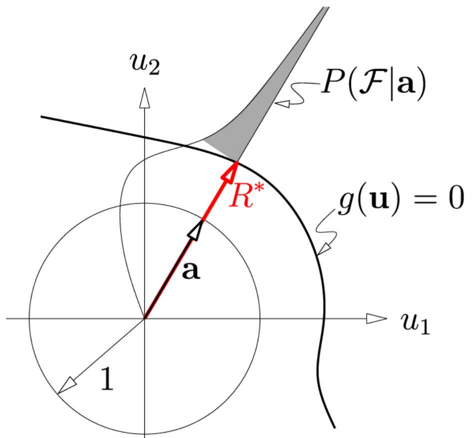
Sigma level	Number of required samples (σ / P_F = 10 %)
$\pm 2\sigma$	2.222
$\pm 3\sigma$	37.037
$\pm 6\sigma$	5.000.000.000

$$\hat{P}_F = \frac{1}{N} \sum_{i=1}^{N} I\left(g(\mathbf{x}_i)\right) \qquad \hat{\sigma}_{P_F} = \sqrt{\frac{\hat{P}_F}{N}}$$

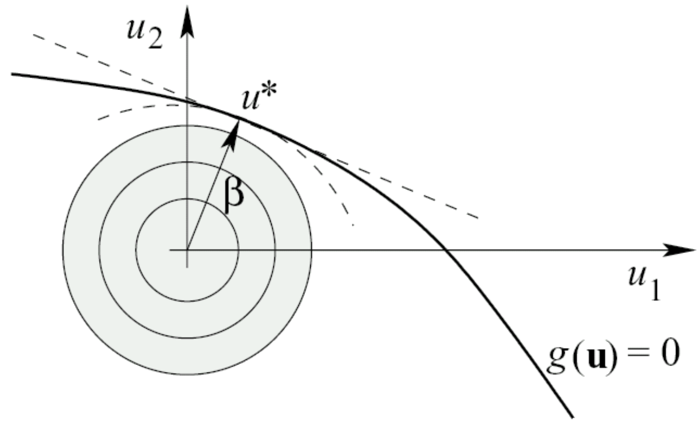
- Robust for arbitrary limit state functions
- Confidence of the estimate is very low for small failure probabilities
- Sigma level ≤ 2
- Independent of number of random variables

Advanced methods for reliability analysis

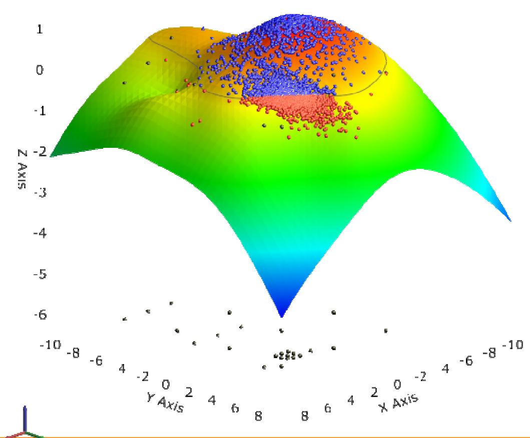
Directional Sampling



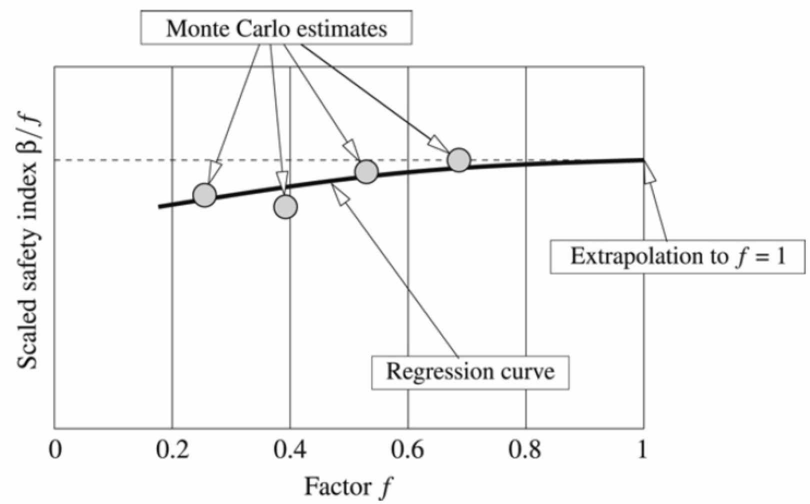
First Order Reliability Method



Adaptive Response Surface Method (Dynardo 2006)



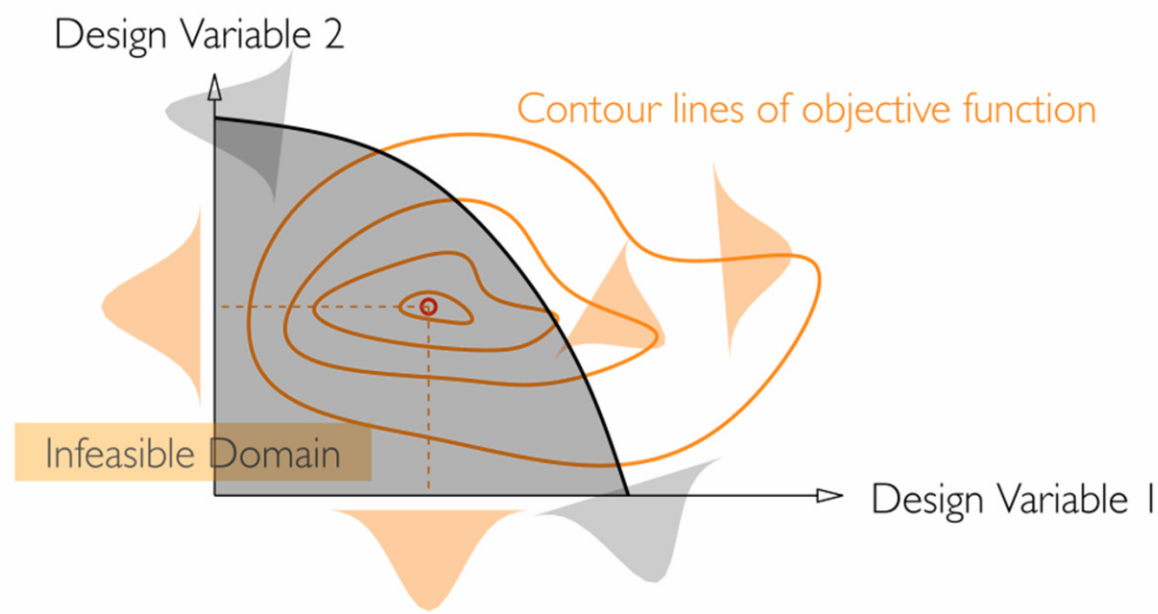
Asymptotic Sampling (Bucher 2009)



Robust Design Optimization

Robust Design Optimization

- Robust Design Optimization (RDO) optimizes the design performance with consideration of scatter of design (optimization) variables as well as other tolerances or uncertainties
- As a consequence of uncertainties the location of the optima as well as the contour lines of constraints scatter



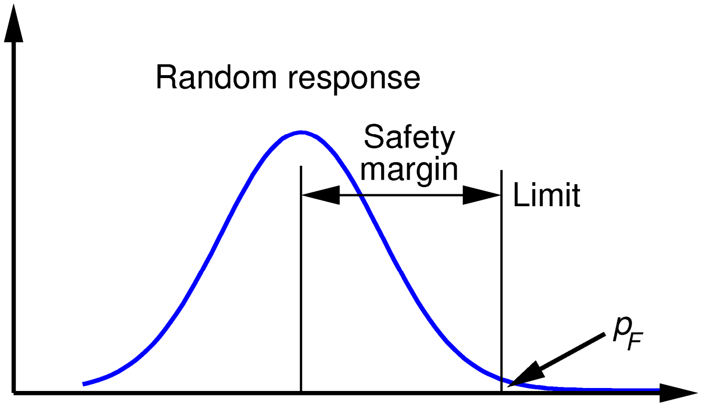
- To proof Robust Designs, safety distances are quantified with variance or probability measures using stochastic analysis

Methods for Robust Design Optimization

Variance-based RDO

- Safety margins of all critical responses are larger than a specified sigma level (e.g. Design for Six Sigma)

$$y_{limit} - y_{mean} \leq a \cdot \sigma_y$$



Reliability-based RDO

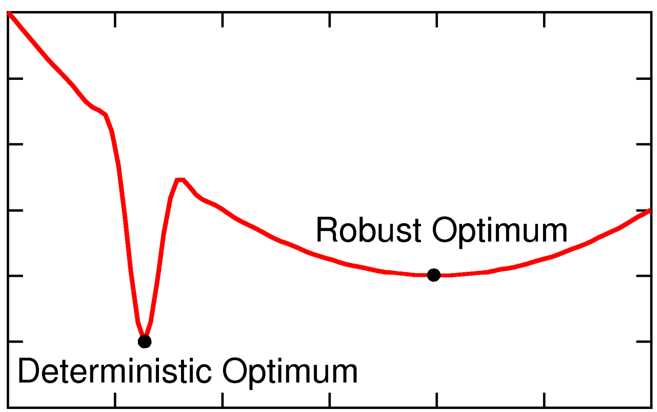
- Failure probability with respect to given limit states is smaller as required value

$$p_F \leq p_F^{target}$$

Taguchi-based RDO

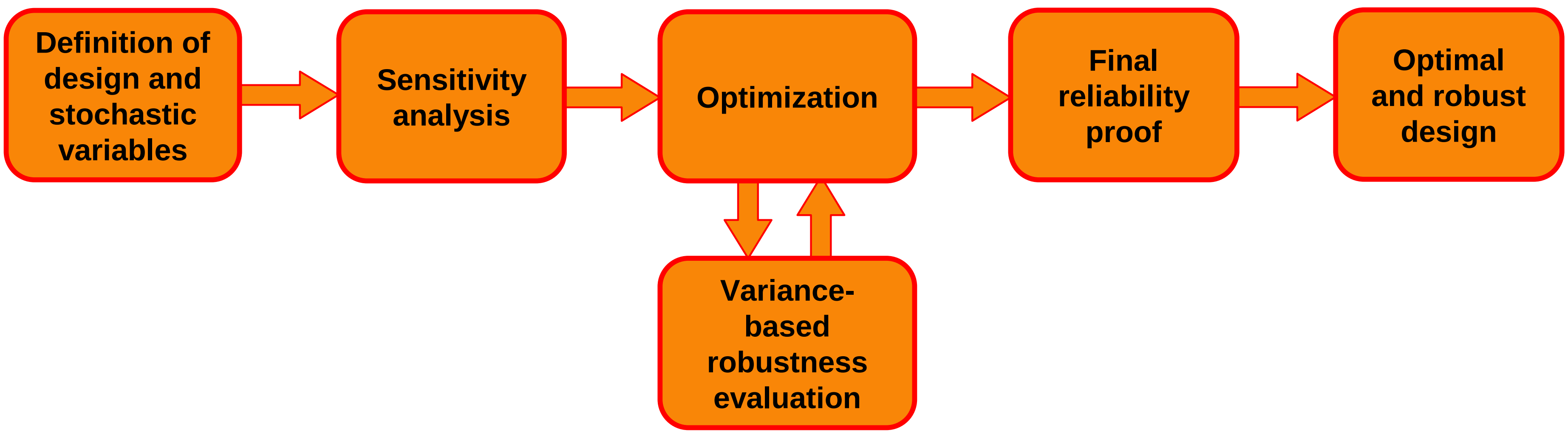
- Taguchi loss functions
- Modified objective function

$$f(y) = \frac{k}{N} \sum y_i^2 = k(\bar{y}^2 + \sigma_y^2)$$



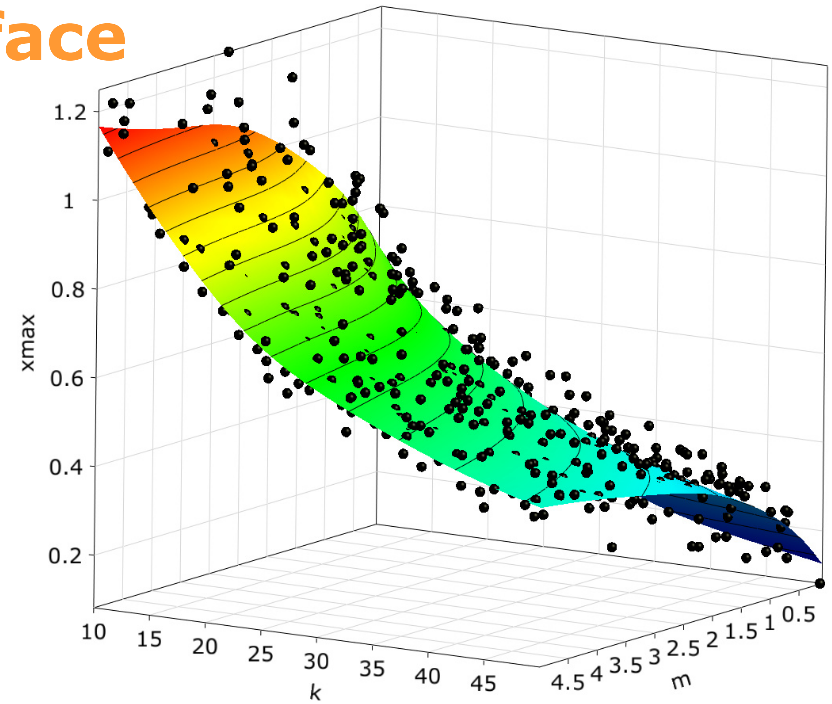
Simultaneous Robust Design Optimization

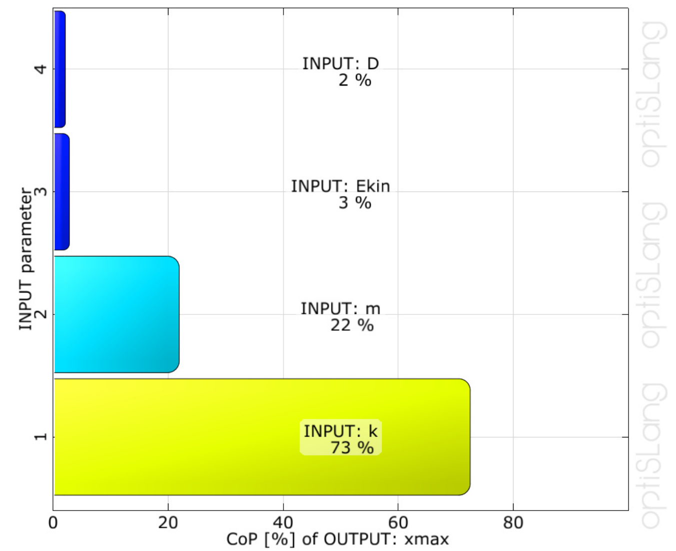
- Fully coupled optimization and robustness/reliability analysis
- For each optimization (nominal) design the robustness/reliability analysis is performed
- Applicable to variance-, reliability- and Taguchi-based RDO
- ➢ Our efficient implementation uses small sample variance-based robustness measures during the optimization and a final (more accurate) reliability proof
- ➢ But still the procedure is often not applicable to complex CAE models



RDO on global response surface

- Approximation of model responses in mixed optimization/stochastic space
- Simultaneous RDO is performed on a global response surface
- Applicable to variance-, reliability- and Taguchi-based RDO
- Approximation quality significantly influences RDO results
- ➢ Final robustness/reliability proof is required
- Pure stochastic variables have small influence compared to design variables
- ➢ Important local effects in the stochastic space may be not represented

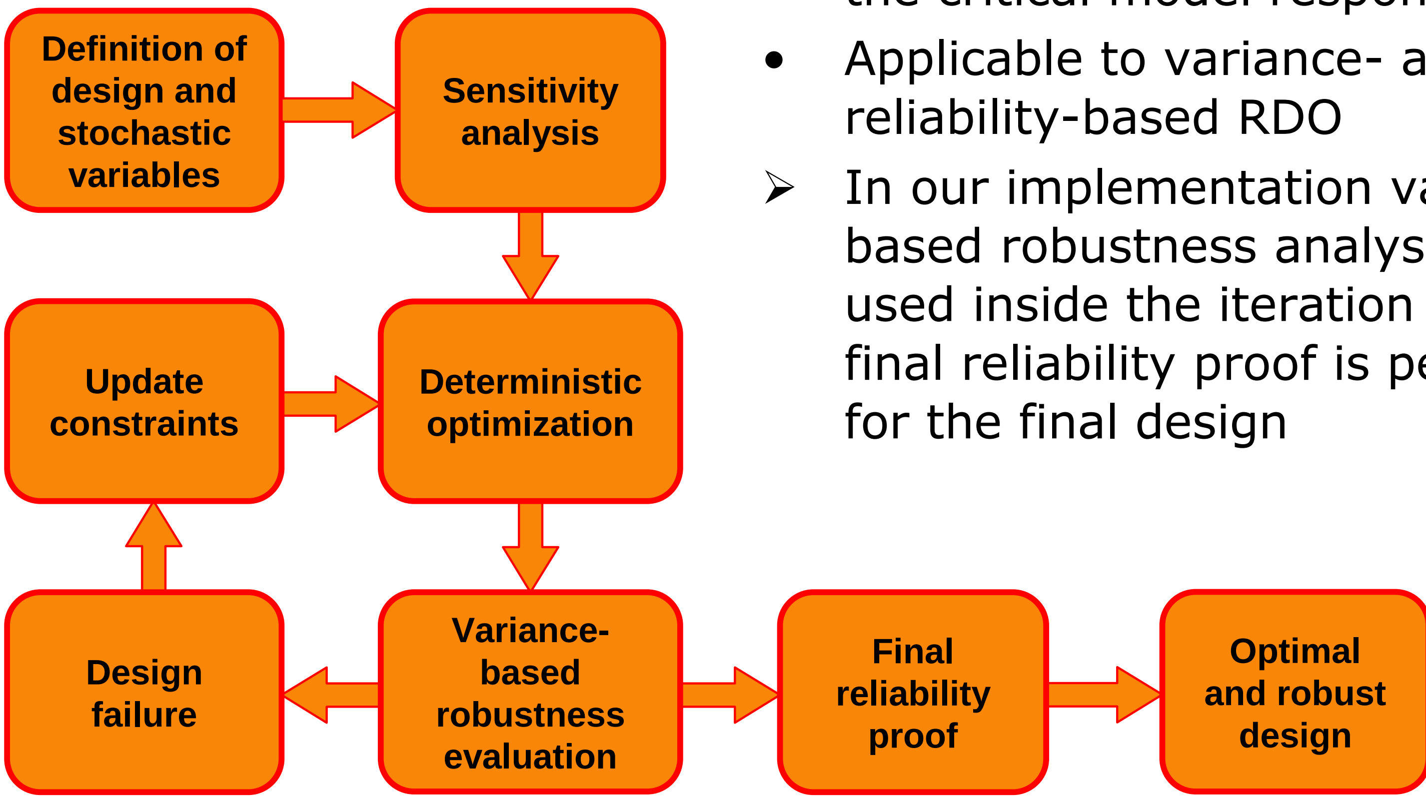




Iterative Robust Design Optimization

- Decoupled optimization and robustness/reliability analysis
- For each optimization run the safety factors are adjusted for the critical model responses
- Applicable to variance- and reliability-based RDO

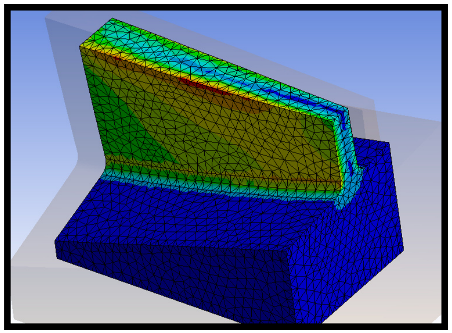
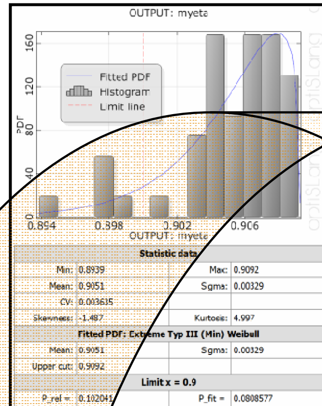
➢ In our implementation variance-based robustness analysis is used inside the iteration and a final reliability proof is performed for the final design



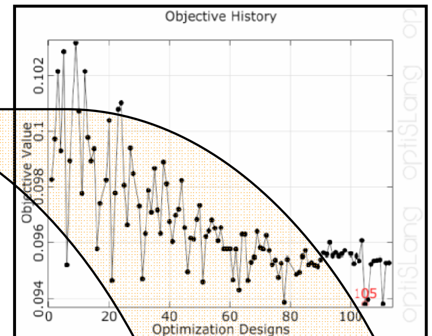
Applications

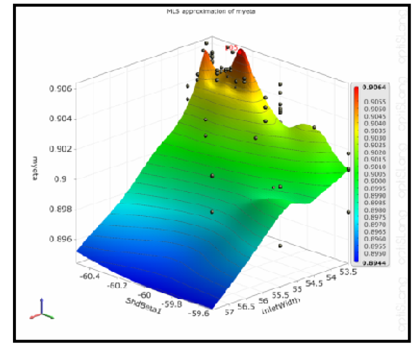
Iterative RDO application - Centrifugal compressor

- RDO with respect to 21 design parameters and 20 random geometry parameters, including manufacturing tolerances
- Robust Design was reached after 400+250=650 design evaluations

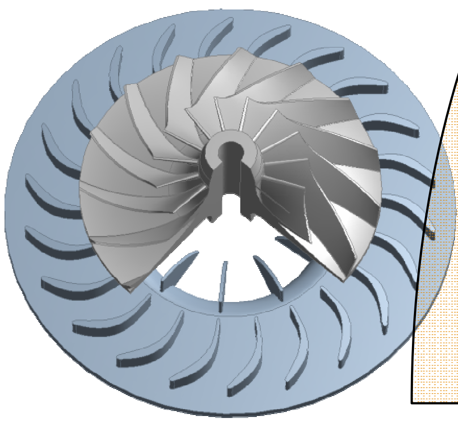
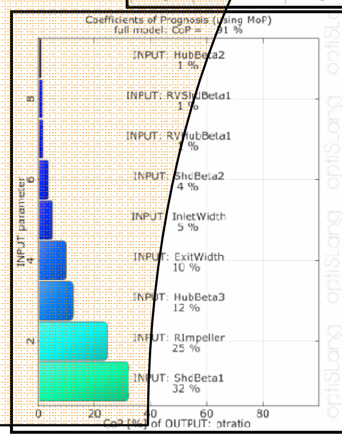



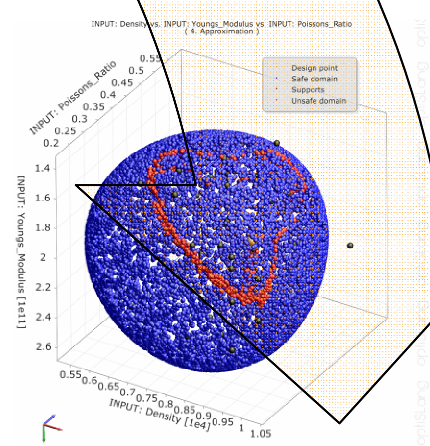
Robustness evaluation

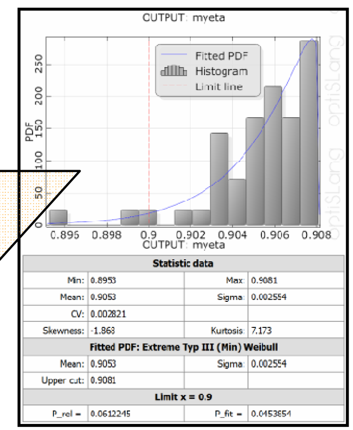




RDO optimization





Sensitivity + first optimization step

Reliability proof

Summary

- Highly optimized structures tend to loose robustness
- Variance-based robustness analysis can estimate small sigma level
- Reliability analysis is necessary to proof small failure probabilities
- Fully coupled optimization and reliability analysis is not applicable to real world problems
- Iterative optimization/variance-based analysis with final reliability proof is applied by Dynardo to industrial problems since several years
- Global response surface approximation may lead to a robust design for sufficient number of support points, but final reliability proof should be performed in any case